Understanding Risk Attitudes in two Dimensions: An Experimental Analysis

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Abstract

Despite extensive studies, the nature of risk attitudes remains as one of the most vigorously discussed questions in economics and psychology. In the framework of expected utility theory, attitude towards risk originates in changes in marginal utility (the curvature of the utility function). Cumulative prospect theory (CPT) adds an additional dimension: the weighting of probabilities. By examining both dimensions, we strive to gain more insights on the nature of risk attitudes: what is the relation between the curvature of utility and probability weighting? How are these related to cognitive limitations? We ran a controlled laboratory experiment to answer these questions. Our findings suggest that the two dimensions capture different characteristics of individual risk attitudes. Though, most individuals are risk averse in both dimensions, the two dimensions show no significant correlation. In addition, only probability weighting is correlated with educational background and decision time. This suggests a relation between the convexity of probability weighting and cognitive limitations.

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1 Introduction

Discussion on the nature of risk attitudes dates back as early as 1713 in a letter by Nicolas Bernoulli, but it remains as a highly controversial topic in economics and psychology till today. In expected utility theory (hereafter EUT), attitudes towards risk originates from the change of marginal utilities (the curvature of utility functions). Wakker (1994) argued that the utility function describes an intrinsic appreciation of money, prior to probability or risk, and that regarding risk attitudes as originating from the perception of probabilities would be more natural since, after all, risk is primarily about the likelihood of outcomes. In line with this argument, prospect theory (Kahneman and Tversky, 1979; Quiggin, 1982) and its more advanced version cumulative prospect theory (Tversky and Kahneman, 1992, hereafter CPT) separated risk attitude into two dimensions: the curvature of utility, and a "probabilistic" component, i.e., the transformation of (cumulative) probabilities.

With this extension, one obtains one more channel through which the nature of risk attitudes can be addressed, since both the curvature of utility and the shape of probability weighting reflect attitude towards risk. What is the relation between them? What is the role of each dimension? Is an individual who is averse in one dimension also likely to be averse in another? Is it possible for an individual to be risk averse in one dimension but risk seeking in another? *What is the nature of risk attitudes?* In this paper, we conducted a laboratory experiment to address these issues.

Several works have elicited the utility function and the probability weighting function. Tversky and Kahneman (1992) themselves conducted an experiment to test for the shape of the utility function and the probability weighting function. Later Wakker and Deneffe (1996) and Abdellaoui (2000) developed the so-called trade-off method to elicit the probability weighting function and utility function separately. In this paper we also use this method. Yet, our focus is different. Apart from measuring and classifying these two functions, we investigate the relation between them and explore the nature of risk attitudes.

There have been several theoretical works discussing these two dimensions of risk attitudes.

Hong et al. (1987) investigated the classification of risk attitudes in Rank dependent utility theory, a special case of CPT. They found that strong risk aversion implies a concave utility function and a convex probability weighting function. In a more recent paper, Schmidt and Zank (2008) investigated similar issues for CPT. They found that in CPT strong risk aversion implies convex probability weighting but not necessarily concave utility.

Several experimental works address the nature of risk attitudes. van de Kuilen (2008) explored experimentally agents' sensitivity towards probabilities. When agents face similar decisions repeatedly with direct feedback on the consequences, the elicited subjective probability weighting function converges significantly towards linearity. In a similar vein, Schunk and Betsch (2006) considered the connection between decision mode and the curvature of the individual utility function. They found that agents in a deliberate decision mode tend to have a nearly linear utility function, while an intuitive decision mode causes the utility function to be more curved. Risk attitudes has also caught the attention of neuro-economists. It has been shown for instance that risk and reward are processed in different parts of the brain, the dorsal and the ventral MPFC respectively (see Xue et al., 2008). Moreover, several studies found a relation between immediate emotions and risky decision making, though the evidence for multiple systems is mixed (see Loewenstein et al., 2008, for a review).

A prerequisite for a better understanding of the nature of risk attitudes is a careful measurement of the two dimensions. Two elicitation methods are common: the parametric and the trade-off method. In this paper we focus on the latter method. While the parametric method provided useful insights about the shape of both functions, it has a serious drawback: the joint fitting of utility and probability weighting, which makes the parameter estimates of these functions interdependent. The trade-off method as developed by Wakker and Deneffe (1996) is so far the only method that allows for a separate measurement of utility and probability weighting. It has been used and further developed by Abdellaoui (2000), van de Kuilen et al. (2007), Abdellaoui et al. (2005), and Kobberling and Wakker (2005). Since our aim is to look at the interplay of the two dimensions, it is crucial to tear utility apart from probability weighting. This makes the trade-off method especially desirable. In the present paper we mostly rely on the version introduced by

Abdellaoui (2000). The detailed procedure is outlined in the following sections.

The paper is organized as follows. Section 2 outlines the method used to obtain utility and weighting functions, i.e., a two-step trade-off methods for utility and weighting functions. Section 3 describes the experimental procedure. The results of this experiment are given in section 4. Finally, section 5 summarizes and discusses the results and implications of the experimental findings.

2 CPT and the trade-off method

Cumulative prospect theory (Kahneman and Tversky, 1992) is a descriptive model for decision making under risk. After 30 years of development, it is now perhaps the most prominent alternative to the expected utility theory *(EUT)*. As opposed to EUT outcomes are evaluated relative to a reference point, and both monetary outcomes and probabilities are evaluated subjectively. In this paper we restrict ourselves to risky prospects involving only gains, i.e. for all prospects that probabilities are known and only nonnegative outcomes are possible. As a result, the reference point can be normalized to zero. For a discussion of the utility function and the probability weighting function over losses see Kobberling and Wakker (2005) and Abdellaoui et al. (2005).

Formally CPT on the gain domain is defined as follows. Let $P = (x_1, p_1; \ldots; x_n, p_n)$ denote a prospect that assigns probability p_i to outcome x_i, where $x_1 < \ldots < x_n$. The evaluation of this prospect depends on two functions: a utility function $u(\cdot)$ and a probability weighting function $w(\cdot)$. The utility function $u(\cdot)$ is assumed to be strictly increasing over the outcome space X, and the function $w(\cdot)$ is a mapping $w : P \to P$, with $w(0) = 0$ and $w(1) = 1$, where $P = [0, 1]$ is the probability space. Finally, the utility of the prospect P is given by:

$$V(P) = \sum_{i=1}^{n} \pi_i^+ u(x_i), \tag{1}$$

where $\pi_i^+ = w^+(\sum_{k=j}^{n} p_k) - w^+(\sum_{k=j+1}^{n} p_k)$ and $\pi_n^+ = w^+(p_n)$.

2.1 The trade-off method

We now demonstrate the detailed procedure of the trade-off method (hereafter TO method). The TO method first elicits a standard sequence of outcomes. This sequence is used to infer the utility function and later serves as a basis for the elicitation of the probability weighting function.

A sequence of outcomes is constructed as follows: Subjects are asked to choose between two lotteries A and B with $A:(p,x_1;1-p,r)$ and $B(p,x_0;1-p,R)$. While x_0, r, R, p are held fixed with $0<r<R<x_0$ and $p\in(0,1)$, x_1 is varied to find an outcome such that subjects are indifferent between the two lotteries. To elicit the next indifference,x_1 replaces x_0 in prospect B and x_2 replaces x_1 in prospect A, and x_2 is varied until subjects are indifferent between the prospect $A:(p,x_2;1-p,r)$ and the prospect $B:(p,x_1;1-p,R)$. In CPT the above two indifference relationships imply

$$[1-w(p)]u(R)+w(p)u(x_0)=[1-w(p)]u(r)+w(p)u(x_1), \tag{2}$$

$$(1-w(p))u(R)+w(p)u(x_1)=[1-w(p)]u(r)+w(p)u(x_2). \tag{3}$$

Combining (2) and (3), it gives

$$u(x_2)-u(x_1)=u(x_1)-u(x_0), \tag{4}$$

that is, the outcomes x_0,x_1,x_2 distribute with equal distance in the utility axis. Repeating this procedure $n-1$ times, we obtain a sequence of outcomes $(x_0,x_1,\cdots,x_n)$ where

$$u(x_{i+1})-u(x_i)=u(x_{i+2})-u(x_{i+1}),\forall i=0,1,\ldots,n-2. \tag{5}$$

Assuming that subjects prefer more money to less, $R>r$ implies $x_{i+1}>x_i$. A small value of x_0 is taken initially and then the elicited outcomes $x_1,\ldots,x_n$ increase stepwise, this way of eliciting utility functions is called the outward TO method. [1]

[1] An alternative method would be to set $R<r$ and with other parameters remaining unchanged. With this modification the sequence of outcomes $x_0,x_1,\ldots,x_n$ decreases, which is called the inward TO method. Fennema and van Assen (1998) compare these two ways of eliciting the utility function and they

With the standard sequence of outcomes $(x_0, x_1, \cdots, x_n)$, we proceed to determine a sequence of probabilities. Again, subjects are asked to choose a lottery

$$\begin{array}{cccccc} Prob. & Gain & & Prob. & Gain & \\ p & x_0 & \sim & p & x_i & \\ 1-p & x_6 & & 1-p & x_i, & \\ \multicolumn{2}{c}{prospect\ A} & & \multicolumn{2}{c}{prospect\ B} & \end{array} \tag{6}$$

here x_0, x_6 are fixed. For each x_i, $i = 1, \ldots, 5$ p_i is varied until an indifference is achieved. This produces a sequence of p_i $i = 1, \ldots, 5$. In CPT, this indifference relationship imply

$$w(p_i)u(x_n) + (1 - w(p_i))u(x_0) = w(1)u(x_i). \tag{7}$$

After some simple algebraic manipulation, we have

$$w(p_i) = \frac{u(x_i) - u(x_0)}{u(x_n) - u(x_0)}, \quad \forall i = 1, \ldots, n-1. \tag{8}$$

By (4), we know that $u(x_{i+1}) - u(x_i)$ is constant for $i = 1, \ldots, n$. Using this condition the above equation can be simplified into

$$w(p_i) = \frac{i}{n}, i = 1, \ldots, n. \tag{9}$$

Going through $i = 1, \ldots, n$, we would have n points for the probability weighting function, of which the weights are calculated as $\frac{i}{n}$.

Several features of the above procedure are worth some additional remarks. Note that, for the elicitation of the utility function few assumptions are needed. Apart from requiring probability weighting function to be positive and increasing in p, no knowledge about the shape of the probability function is needed. This is a substantial advantage compared to the parametric method, where a specific form for the probability functions needs to be assumed. Second, when eliciting probability weighting functions, we only rely on the property of equal distance in utility space for the points $(x_0, x_1, \cdots, x_n)$; no assumption

find that the outward TO method produces results more consistent with Tversky and Kahneman (1992) and Abdellaoui (2000). In order to produce results comparable with previous literature, we also employ the outward TO method.

about the form of the utility function is needed. Thus, the above procedure effectively avoids the confounding problem resulting from the simultaneous elicitation of the utility function and the probability weighting function in the parametric method. As a drawback of the TO method it should be mentioned that the reliability of the measurement of probability weighting depends crucially on the accurate assessment of the utility function.

3 The experiment

3.1 Subjects, procedure, and payment

The experiment was conducted in June 2008 with 124 Jena university undergraduate students. Among those 37 subjects study natural science and economics, e.g., physics, mathematics, economics, and business administration. In total we ran 4 sessions. Each session lasted about 50 minutes. Altogether the experiment consisted of 4 parts. In this paper we will only present the first two parts that concern the elicitation of the utility function and the probability weighting function.[2] As explained in the above section, we first constructed a standard sequence of outcomes (hereinafter TO experiment), and then used this standard sequence of outcomes to elicit a sequence of probabilities (hereinafter PW experiment). Each part consisted of several rounds (42 rounds for the TO and about 32 rounds for the PW part), out of each part one round was individually selected at random, the preferred lottery was played and results paid to the participant. Instructions were handed out and read out aloud and questions were answered privately, an English translation of the original instructions is attached in the appendix. The average earning in the experiment was 16 Euros. The results of the two other parts will be reported in a different paper. The experiment was programmed with ztree (Fischbacher (2007)). Participants' invitation was managed by ORSEE (Greiner (2004)).

We used the TO method to elicit the utility and the probability weighting function separately. All outcomes and probabilities were obtained through a series of choice questions.

[2]The results of the last two part of the experiment are discussed in Qiu and Steiger (2009)

Each question consisted of a choice between two prospects, and subjects were asked to choose the prospect they prefer.

3.2 Eliciting a standard sequence of outcomes for utility functions: TO experiment

In the TO experiment we set: $p = \frac{1}{2}$, $r = 0$, $R = 10$, and $x_0 = 20$, eliciting in total a sequence of 6 outcomes $x_1, x_2, \ldots, x_6$. Given a known gain x_i, x_{i+1} was varied to establish the following indifference relationship:

$$\begin{array}{cc} \textit{Prob.} & \textit{Gain} \\ 0.5 & x_{i+1} \\ 0.5 & 0 \\ \multicolumn{2}{c}{\textit{prospect A}} \end{array} \sim \begin{array}{cc} \textit{Prob.} & \textit{Gain} \\ 0.5 & x_i \\ 0.5 & 10. \\ \multicolumn{2}{c}{\textit{prospect B}} \end{array} \tag{10}$$

While the concept of arriving an indifference relationship is clear, the practical implementation is not straightforward. For implementation some studies rely on the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al., 1964), see e.g. Irwin et al. (1998) and Keller et al. (1993), others rely on the auction method see, e.g., Coppinger, Smith, and Titus, 1980; Cox, Roberson, and Smith, 1982; Kagel, Harstad, and Levin, 1987; and Kagel and Levin, 1993. Both methods ask subjects to pick their indifference value out of a given range. A sensible choice, however, involves a through understanding of the mechanism, and choosing a value out of a continuous range is typically cognitive demanding. Noussair et al. (2004) suggest that subjects are often confused or do not taken the procedure seriously. They show experimentally that compared to other methods, the choice based method is easier for subjects to understand, and consequently yields more reliable data. Choice based methods, however, have one obvious drawback: the large number of choices required to make good inference may exhaust participants' concentration. Taking above considerations into account, we rely on a (modified) bisection choice procedure. The detailed algorithm of the (modified) bisection choice procedure is given in the Appendix 1.

After the elicitation of the six outcomes $x_1, x_2, \ldots, x_6$, we checked for consistency of choices. Subjects were again presented with the 7^{th} iteration of each x_i. Note, that although this additional pair of choices has not reached the final indifference relationship, the remaining interval is already quite small. This makes the consistency check a rather tough test.[3]

3.3 Eliciting the probability weighting function: PW experiment

Having elicited a sequence of x_i, we proceeded to the sequence of probabilities. , $p_1, ..., p_5$. Subjects were presented with pairs of prospects of structure (6), $(x_0, p_i; x_6)$ and (x_i). Here p_i was varied to establish a indifference relationship. Again the indifference relationships were established via a (modified) algorithm.

1. For each p_i we first presented subjects with a fixed sequence of five pairs of prospects of structure (6), where p_i is successively set to $.1, .9, .3, .7, .5$. Having elicited those sequences for all $x_i, i = 1, \ldots, 5$, we proceeded with the bisection procedure.
2. If there was only one switching point, two further iterations would be employed to find the point of indifference. For instance, if for a given x_i a subject preferred B over A for $p_i = 0.3$ and A over B for $p_i = .5$, then it could be inferred that her indifference probability must lie within the interval $[.3, .5]$. The bisection procedure (proceeding with $p_i = .4$) would be applied two times to elicit the indifference probability for this x_i.
3. If there were two or more switching points, a interval encompassing all switching points would be determined and a maximum of 4 iterations of the bisection procedure was employed to find out the indifference probability.

Having elicited the sequence of probabilities $p_1, p_2, \ldots, p_5$, we checked for consistency by eliciting indifference between (x_3) and $(x_4, p_6; x_2, 1 - p_2)$. According to CPT choices are consistent if $p_6 = p_3$.

[3] Given the number of choices a memorization of choices seems rather unlikely.

4 Results

We report the results in two steps. We start with some general results for utility and probability weighting, proceed with the classification of them in terms of risk attitudes, and finally turn to our main result: the relationship between these two dimensions of risk attitudes. For the analysis we had to discarded 5 out of 124 subjects, partly due to computer problems and partly due to insufficient sensitivity towards the stimulus.

4.1 Classification of utility functions

To check for consistency in participant's choices the 7^th choice pair of each x_i was repeated. Preference reversal occurred in 30% of the cases. Though this number may seem large, note that the remaining interval for the inference of x_i at the 7th choice is already quite small. Thus a consistent choice suggests strongly data reliability whereas a inconsistent choice does not necessarily imply a poor decision. The value is also comparable to the findings in Starmer and Sugden (1989) (26.5%) and Camerer (1989) (31.6%), which suggests that the elicited x_i are rather reliable.[4]

We classified the participants' utility or value function using $u(x) = x^{\alpha}$, which is often used in the literature. It may seem to be surprising that we favor the TO method over parametric fitting, but still fit a power utility function. Notice however that our purpose is not to obtain a precise α for each individual, rather we are only interested a ranking of concavity of utility functions among subjects. The estimated αs should provide enough information. The sequence of values, $x_1, x_2, ..., x_6$ enables us to estimate α for each subject. An $\alpha < 1$ implies a concave utility function, while $\alpha \approx 1$ implies a linear utility function, and $\alpha > 1$ implies convex utility function. For a linear utility we set a tolerance level of $0.9 < \alpha < 1.1$. According to our classification 67 subjects have concave ($\alpha < 0.9$) , 24 subjects have linear ($0.9 < \alpha < 1.1$), and 28 subjects have convex ($\alpha > 1.1$) utility

[4]Note that for x_1, when the interval is rather small preference reversal occurs in 39% of the cases, while it lowers to 23% for x_6. This further emphasises that preference reversal was a result of the rather small choice interval.

	utility		probability weighting	
	α	Difference	γ	Difference
concave	67	68	91	81
linear	24	16	5	34(15+19)
convex	28	15	23	4

Table 1: Classification of utility and probability weighting, first according to parametric fitting and second to the difference method.

functions. We varied this tolerance level slightly and found the results to be robust. Figure () displays the distribution of α.

Since a wrong choice of parametric specification may bias results, we additionally used the non-parametric difference method to check for robustness of the above classification. We calculated the first order difference $\Delta'_i = |x_i - x_{i-1}|$ for $i = 1, ..., 6$ and the second order difference $\Delta''_j = \Delta'_{j+1} - \Delta'_j$ for $j = 1, ..., 5$. Similar to Abdellaoui (2000), we classify

- a utility function to be concave if $\Delta''_j > 0$ for more than 3 out of 5 times,
- a utility function to be convex if $\Delta''_j < 0$ for more than 3 out of 5 times, ,
- a utility function to be linear.if $\Delta''_j \approx 0$ for more than 3 out of 5 times,

With these criteria, we classified 68 subjects as concave, 16 as linear, and 15 subjects as convex utility. The remaining 20 subjects could not be classified with this method. As shown in table (1), the two classification methods yield rather similar results. Hence, α reasonably captures the shape of the utility function.

4.2 Classification of probability weighting functions

To increase reliability of the data, we checked for consistency by comparing $(x_6, p_3; x_0) \sim (x_3)$ and $(x_4, p'_3; x_2) \sim (x_3)$. According to CPT, the two probabilities should be the equal

($p_3 = p'_3$). Indeed, both median values of p_3 and of p'_3 are equal to 0.5, and they are not significantly different (the mean difference $p_3 - p'_3 = -0.015$, $p > 0.10$).

To confirm the non-linearity of probability functions, we performed a Friedman-test. The hypothesis that the probability weighting function is linear can be rejected at the 5% of level ($\chi^2 = 15.9137$ with $p < .0031$).

A universal classification of the probability weighting function is difficult. Previous experiments find S, inverse S, linear, S, convex, as well as concave shaped probability weighting functions. The shape of the probability weighting has an important impact on risk attitudes. For instance, when lotteries involving only two outcomes and utility function plays no role, an inverse S shaped probability weighting function implies risk aversion for large probability gains and risk seeking behavior for small probability gains, while an S shaped probability weighting function implies the opposite. A convex probability weighting function implies risk aversion for gains, while a concave probability weighting function implies risk seeking for gains.

To properly classify probability weighting functions, we first checked each subject's array of p_i for patterns. We found that the vast majority of subjects has a convex probability weighting pattern. The non-parametric difference method confirms this. Note that the pattern of probability weighting is best discovered when p is close to 0 or 1, where probability weighting is suspected to be most severe, while the middle range, i.e., when p is close to 0.5, patterns may be less obvious. Thus a crude but simple way to check for the shape of probability weighting functions is to compare the pairs ($w_1 \sim p_1$) and ($w_5 \sim p_5$). A convex probability weighting function implies $w_1 < p_1$ and $w_5 < p_5$, while a concave probability weighting function implies $w_1 > p_1$ and $w_5 > p_5$, an inverse S-shaped probability weighting function implies $w_1 > p_1$ and $w_5 < p_5$, and finally an S-shaped probability weighting function implies $w_1 < p_1$ and $w_5 > p_5$. Based on these criteria, we classified 81 subjects as pessimistic, 4 subjects as optimistic, 19 subject as inverse S-shaped, and 15 subjects as S-shaped.

This finding is not unusual comparing to previous literature. van de Kuilen (2008) and van de Kuilen et al. (2007) also found that majority of subjects possess a convex probability weighting function and there was little evidence for inverse S shaped probability weighting functions. Recall that in both Hong et al. (1987) and Schmidt and Zank (2008), a convex, concave or linear probability weighting function corresponds to risk aversion, risk seeking, or risk neutrality in this dimension. Also as discussed above, subjects with an inverse S or S shaped probability weighting are risk averse at some probabilities but risk seeking at other probabilities, which makes a consistent classification of subjects in terms of risk attitudes difficult. To discuss the relationship between the two dimensions of risk attitudes, we need to obtain a more precise classification of probability weighting. This measure should consistently classify subjects' risk attitudes in the probability dimension. van de Kuilen et al. (2007) found that the power-family functions yielded the best fit, even better than the families with extra parameter do. Since our data shows similar pattern, we assume a probability weighting function with the shape $w(p) = p^{\gamma}$. With five data points $p_i, i = 1, 2, 3, 4, 5$, we estimated a γ for each subject. One might object that we use parametric fitting although we recommend a non-parametric method. It should be emphasized that we are not interested in the value of γ per se. What we need is only a clean ranking of *the convexity of probability weighting functions*, which so far only the TO method allows since it avoids the joint fitting of utility and probability.

In order to highlight the different dimensions of risk attitudes, we classify the probability weighting function as follows:

- <u>concave/optimistic</u>: a subject is optimistic if her probability weighting function is concave ($\gamma < 1$),
- <u>linear/neutral</u>: a subject is neutral if her probability weighting function is linear ($\gamma \approx 1$), and
- <u>convex/pessimistic</u>: a subject is pessimistic if her probability weighting function is convex ($\gamma > 1$).

Again, we varied the tolerance level for γ, and the classification result was robust. We fixed the range for linear probability weighting to $0.95 < \gamma < 1.05$. According these criteria, we classified 91 subjects as pessimistic, 23 subjects as optimistic, and 5 subject as linear.

	concave α	linear α	convex α	sum
pessimistic γ	52	18	21	91
neutral γ	1	2	2	5
optimistic γ	14	4	5	23
sum	67	24	28	119

Table 2: The two dimensions of risk attitudes

Note, that these results are similar comparable to the non-parametric difference method.

4.3 Central results

Last we turn to our main hypothesis: What is the nature of risk attitudes? More specifically, what's the relationship between the two dimensions of risk? How are they related with cognitive limitations.

We first present the relationship between the two dimensions of risk attitudes. The results are reported in Table (2).

The largest group in Table (2) are the subjects with concave utility functions and pessimism in the probability weighting dimension (52 subjects). This finding is amiable to economists, since most theoretical models rely on the assumption that agents are risk averse. Our result suggests that the majority of the population may indeed be risk averse in both dimensions

There are further interesting patterns in the data. The third cell in the first row denotes the convex/pessimistic subjects. They are the second largest group in our classification (21 subjects). Mirroring this is the first cell in the third row. This cell denotes the concave/optimistic subjects. Here we have 14 subjects. These subjects are risk averse in one dimension but risk seeking in the other. This is interesting since although both utility functions and probability weighting functions captures information about risk attitudes, they seem to have different foundations.

The subjects who are risk averse in both dimension represent the largest proportion. Among these subjects, one natural question to ask will be: is a subject who is more risk averse in one dimension is also more likely to be risk averse in the other? If this is so, these two dimensions of risk attitudes are well correlated, and then it might not be that problematic to use the curvature of utility function as the single proxy for risk attitudes. To test this hypothesis, we ran a Spearman's ρ rank correlation test between α and γ for these 53 subjects. The correlation is insignificant (Spearman's ρ, $p > 0.10$). This finding suggests that these two dimensions of risk are different and, therefore, necessary to consider both.

A more general illustration of our main result is shown in figure **??**, here for each individual participant the relation between alpha and gamma is plotted. The x-axis depicts alpha and the y-axis the gamma. The rectangles correspond to the labeling in table 2, with the upper left rectangle depicting the concave & pessimistic, the upper mid square the neutral pessimistic subjects etc. Note that in order to produce a more condensed picture the graph is limited to subjects with $\alpha < 1.5$ and $\gamma < 2$. Though most observations are in the upper left square of the graph, it can be seen that dots are evenly distributed with no apparent pattern or piling.

These results raise questions on the nature of risk attitudes. Schunk and Betsch (2006) argue that risk attitudes result from cognitive limitations. They found that people with large cognitive bias also exhibit higher risk aversion. If risk attitude results from cognitive limitation, education may play an important role because either some fields offer more trainings in the handling of risk and probabilities or because people who are less cognitively constrained are more willing to choose fields offer such trainings. We examine subjects' field of study in comparison to risk attitudes via two linear regressions. In the first regression the dependent variable is subjects' α, in the second regression the dependent variable is subjects' γ. Explanatory variables are field of study and gender in both regressions. The field of study is classified into two categories. The comparison category (Major 1) is natural science and economics, e.g., physics, mathematics, economics, and

Dep. Variable	Expl. Variable	Coefficient	Std Error	t-statistic	p-value
α	Intercept	0.8890	0.0338	26.305	< 0.01**
	Gender	0.0839	0.0565	1.486	0.140
	Major 1	0.0085	0.0556	0.153	0.879
γ	Intercept	1.8860	0.1408	13.397	< 0.01**
	Gender	-0.0608	0.2355	-0.258	0.7968
	Major 1	-0.3870	0.2316	-1.671	0.0975*

** 5% significant level, and * 10% significant level

Table 3: Results of the two linear regressions

business administration. We have 37 subjects in this category. Students with other majors are classified into the reference category. We have 86 subjects in this category. Gender is a dummy variable, where female is 0 and male is 1. The results of the regression are reported in Table (3).

The concavity of utility seems rather stable. Education does not show a significant influence (p value equals to 0.879). In contrast, the convexity of probability weighting is significantly influenced by education. Students studying natural science and economics exhibit much less convexity than students with other majors. One possible explanation for this observation is that the concavity of utility functions is a more fundamental trait, and it does not originate from cognitive limitations; whereas the convexity of probability weighting originates from cognitive limitations, and it can be "corrected" with proper trainings. This result is comparable with Van de Kuilen and Wakker (2006) and van de Kuilen (2008) where they demonstrate that the weighting of probabilities is significantly diminished if subjects are given regular feedback and have the opportunity to learn. We did not find a significant difference of risk attitudes between males and females.

Finally, we investigate the relationship of decision time, measured by the mean of the last choice for each x_i and p_i, and both dimensions of risk attitudes. This relation, however, is not straightforward. One can argue both ways: on the one hand, taking more time for decision making implies a more deliberate decision and, therefore, *using* more decision time indicates less bias and consequently less risk aversion. The reverse, a negative relation also is possible: *needing* more decision time indicates less cognitive capacity and, therefore, corresponds to more risk aversion. These two opposing arguments suggest that a clean relation between decision time and risk attitudes is best found on a relatively more homogeneous pool. Performing the analysis on the whole subject pool confirms the above intuition. We found no correlation between decision time and risk attitudes in either dimension.

Looking at table (2) we conjecture that subjects in the same cell may have more similar characteristics than the group of all subjects and are thus more homogeneous. We performed a correlation analysis on the subjects who are risk averse in both dimensions. We found that the decision time was positively correlated with the convexity of probability weighting functions (Spearmann correlation test, $p < 0.05$), but not with the concavity of utility functions. This again suggests that the weighting of probabilities is indeed due to cognitive limitations. Subjects who needed more time to make a decision are more likely to have high cognitive limitation and are therefore more likely to distort probabilities.

5 Discussion

It is now probably less controversial to argue that risk attitudes have two dimensions. Yet, to the best of our knowledge no study so far looked at the relation between these two dimensions of risk and relating them with cognitive limitations. This paper serves to answer this question. Our result suggests that the two dimensions of risk attitudes capture different characteristics of individuals' risk attitudes. Although most individuals are risk averse in both dimensions, the two dimensions show no significant correlation. Hence, an accurate appreciation of risk attitudes requires the measurement of both. Predictions only based on the curvature of utility functions can be quite far from real behaviors, as showed by the findings in numerous literature.

A deeper understanding of the two dimensions of risk attitudes and their interplay helps to gain insights on the nature of risk attitudes. Our results suggest that individual attitude towards utility seems to be a stable, inherent trait. Weighting of probabilities, on the other hand, is less stable and can be affected by experience and education. One possibility to further understand the two dimensions of risk and their interplay may be to elicit more data. In particular, to test for stability in the two dimensions and compare utility and probability weighting at different utility levels. Another way to examine the nature of risk attitudes via the two dimensions might be to use brain scanning. It is highly likely that decision making under certainty activates different parts of brain than decision making under risk does since the former situation involves only the utility function, while the latter involves both.

6 Appendix 1: the (modified) bisection choice procedure

The detailed algorithm of the (modified) bisection choice procedure is as follows:

1. Given x_i, we set a range for x_{i+1}'s indifference value. This range should be large enough to include potential indifference values for x_i, and it should be small enough to allow for a good inference of the indifference point. We used the following equation to determine this potential range was determined by the following equations:

$$\underline{x} = max\{0, (x_i + R) * 0.5 - r\} \quad (11)$$

$$\bar{x} = (x_i + R) * 1.5 - r. \quad (12)$$

   The determination of this range reflects the combined consideration of flexibility and efficiency. Let $x_m = \frac{\underline{x}+\bar{x}}{2}$ denote the middle point of the interval $[\underline{x}, \bar{x}]$. Subjects were first presented a pair of lotteries as in (10), with $x_{i+1} = x_m$. To ease calculations only integers were allowed. When x_i is not a even integer, the closest even integer larger than x_i is taken.

2. If A is preferred, we know that x_{i+1} must be increased in order to achieve indifference. We thus let $x_{i+1} = \frac{x_m+\bar{x}}{2}$. Likewise, if B is preferred, x_{i+1} must be decreased. We then let $x_{i+1} = \frac{x_m+\underline{x}}{2}$.

3. Repeating this procedure 4 more times, the interval containing the indifference point will become rather small. Finally, we choose the middle point of the final interval to be x_{i+1}.

A drawback of the bisection procedure is that it may not be entirely incentive compatible. If subjects are aware of the entire experimental procedure from the start, they may have an incentive to strategically misreport their choices. To see this, note that pretending to be overly risk averse, i.e. choosing A all the time, raises x_{i+1} and thus increases the mean payoff of prospects B. Since subjects are paid their preferred prospect in one randomly chosen pair, this misreporting strategy may increase their expected experimental payoff. To make it more difficult to fully grasp the bisection procedure, we added two choices at the beginning elicitation procedure. Therefore, in total eight choices were taken to elicit each point. The display of these two choices is independent from participant's choices and is expected to make the inference of the whole algorithm more difficult.

The procedure may be best understood with a numerical example. In the experiment we started the elicitation with the following pair of prospects: $A = (20, 0.5; 10) \sim B = (x_1, 0.5; 0)$. The potential range of x_1 is $[15, 45]$. Participants will then face the following sequence of choices.

No.	Alternatives	Choice	Inference
1	$A = (20, 0.5; 10)$ vs $B = (30, 0.5; 0)$	A	$x_1 \in [30, 45]$
2	$A = (20, 0.5; 10)$ vs $B = (24, 0.5; 0)$	A	$x_1 \in [30, 45]$
3	$A = (20, 0.5; 10)$ vs $B = (38, 0.5; 0)$	A	$x_1 \in [38, 45]$
4	$A = (20, 0.5; 10)$ vs $B = (34, 0.5; 0)$	A	$x_1 \in [38, 45]$
5	$A = (20, 0.5; 10)$ vs $B = (41, 0.5; 0)$	B	$x_1 \in [38, 41]$
6	$A = (20, 0.5; 10)$ vs $B = (39, 0.5; 0)$	A	$x_1 \in [39, 41]$
7	$A = (20, 0.5; 10)$ vs $B = (40, 0.5; 0)$	A	$x_1 \in [40, 41]$
8	$A = (20, 0.5; 10)$ vs $B = (41, 0.5; 0)$	B	$x_1 \in [40, 41]$

Based these choices, x_1 is set to equal to the middle point of the final range $[40, 41]$, that is, 40.5. If subjects choose A all the way, we simply set x_1 equal to the upper bound of the initial range, which is 45.[5]

7 Appendix 2: Experimental Instructions

7.1 General Information

Thank you for participating in our experiment. Please end all conversations now and switch off your cell phone. Please read the instruction carefully. The money you earn will

[5]For the current example one may find 8 choices are too much. For later rounds, this will be necessary since x_i increases with sequence and so does the potential range of x_i.

depend on the choice you make. The money will be paid to you in cash at the end of the experiment. Throughout the experiment, we shall speak of ECU (experimental currency units) rather than Euro. The exchange rate between ECU and Euro is fixed to

20 ECU= 1 Euro Please do not communicate during the experiment, and raise your hand if you have questions. We will answer your questions individually. It is very important that you obey these rules, since we would otherwise be forced to exclude you from the experiment and hence from payment.

The Experiments consists of four parts. Each part consists of several rounds. In each round you have to make a decision. At the end of the experiment one round of each part is selected for payment. In all four rounds will be relevant for your payment.

7.2 Instructions for the TO experiment

The first part of the experiment comprises 42 rounds. In each round, you will be presented with a pair of risky alternatives. Your task is to pick your preferred alternative. To make the comparisons easier, the payoffs are also presented in the upper right corner of the screen. The pairs of risky alternatives will have the following format:

The alternatives shown above can be better understood by using the following thinking. Imagine a big watch with one arm. In above figure, 40% of the panel is covered by white and 60% of the panel is covered by black. The arm of the watch stops equally likely at each position of the watch. Suppose now you have chosen alternative A from the above pair. Then, if the arm stops in the white area, you are paid 300 ECU, if the arm stops at the black area, you are paid 100 ECU. (Equivalent, had you chosen B you would be paid 200 in case of black and 50 in case of white)

At the end of this part of the experiment, one of your choices will be randomly selected and played, and the resulting outcome will be your experimental earning in this part.

7.3 Instructions for the PW experiment

This part is similar to the first part. Again you will be asked for your preference between two lotteries, the difference being that lottery B always gives a fixed payoff. Another difference is that the probabilities in lottery A change for each decision. Using the picture of the first part: the division of the circle between black and white changes for each decision. Please think carefully before each decision, since a confirmed choice cannot be changed.

References

Abdellaoui, M. (2000). Parameter-free elicitation of utility and probability weighting functions. *Management Science*, 46(11):1497–1512.

Abdellaoui, M., Vossmann, F., and Weber, M. (2005). Choice-based elicitation and decomposition of decision weights for gains and losses under uncertainty. *Management Science*, 51:1384–1399.

Becker, G. M., Degroot, M. H., and Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral Science*, 9:226–232.

Camerer, C. F. (1989). An experimental test of several generalized utility theories. *Journal of Risk and Uncertainty*, 2:61–104.

Fennema, H. and van Assen, M. (1998). Measuring the utility of losses by means of the tradeoff method. *Journal of Risk and Uncertainty*, 17(3):277–95.

Fischbacher, U. (2007). z-tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics*, 10(2):171–178.

Greiner, B. (2004). The online recruitment system orsee - a guide for the organization of experiments in economics.

Hong, C. S., Karni, E., and Safra, Z. (1987). Risk aversion in the theory of expected utility with rank dependent probabilities. *Journal of Economic Theory*, 42(2):370–381.

Irwin, J. R., Mcclelland, G. H., Mckee, M., Schulze, W. D., and Norden, N. E. (1998). Payoff dominance vs. cognitive transparency in decision making. *Economic Inquiry*, 36(2):272–85.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):263–292.

Kahneman, D. and Tversky, A. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5:297–323.

Keller, L. R., Segal, U., and Wang, T. (1993). The beckercdegrootcmarschak mechanism and generalized utility theories: Theoretical predictions and empirical observations. *Theory and Decision*, 36:83–97.

Kobberling, V. and Wakker, P. P. (2005). An index of loss aversion. *Journal of Economic Theory*, 122(1):119–131.

Loewenstein, G., Rick, S., and Cohen, J. D. (2008). Neuroeconomics. *Annual Review of Psychology*, 59:647–72.

Noussair, C., Robin, S., and Ruffieux, B. (2004). Revealing consumers' willingness-to-pay: A comparison of the bdm mechanism and the vickrey auction. *Journal of Economic Psychology*, 25(6):725–741.

Qiu, J. and Steiger, E.-M. (2009). Relating risk attitudes and probability updating: An experimental study. mimeo.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior & Organization*, 3(4):323–343.

Schmidt, U. and Zank, H. (2008). Risk aversion in cumulative prospect theory. *Management Science*, 54(1):208–216.

Schunk, D. and Betsch, C. (2006). Explaning heterogeneity in utility functions by individual differernces in decision modes. *Journal of Economic Psychology*, 27(3):386–401.

Starmer, C. and Sugden, R. (1989). Violations of the independence axiom in common ratio problems: An experimental test of some competing hypotheses. *Annals of Operations Research*, 19:79–102.

Tversky, A. and Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5(4):297–323.

van de Kuilen, G. (2008). Subjective probability weighting and the discovered preverence hypothesis. *Theory and Decision*, forthcomming.

Van de Kuilen, G. and Wakker, P. (2006). Learning in the allais paradox. *Journal of Risk and Uncertainty*, 13:155–164.

van de Kuilen, G., Wakker, P., and Zou, L. (2007). A midpoint technique for easily measuring prospect theory's probability weighting. CREED, University of Amsterdam, The Netherlands.

Wakker, P. (1994). Separating marginal utility and probabilistic risk aversion. *Theory and Decision*, 36:1–44.

Wakker, P. and Deneffe, D. (1996). Eliciting von neumann-morgenstern utilities when probabilities are distorted or unknown. *Management Science*, 42(8):1131–1150.

Xue, G., Lu, Z., Levin, I. P., weller, J. A., Li, X., and Bechara, A. (2008). Functional dissociations of risk and reward processing in the medial prefrontal cortex. *Cerebral Cortex*, 10:1–9.