Long-Term Contracts, Irreversibility and Uncertainty

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Long-term contracting implies contracting based on expected future demand or a realization of an uncertain future demand. In this paper I develop a model with asymmetric information on the agent's cost where the initial contract is signed ex ante and relies on an uncertain future surplus function. Irreversible initial investments can be supplemented according to the true surplus generated by the production. This paper distinguishes situations with add-on investments from situations with once-and-for-all investments. Intuitively, with the possibility of additional investments, the first-period investment will be be lower than when no additional investment can be implemented to fit the true demand. This paper affirms this intuition and characterizes the level of investment. Further, asymmetric information distorts the level of investment from its first-best level. When uncertainty is unverifiable, I show that the distortion is stronger under complete contracting than under incomplete contracting. When uncertainty is verifiable, distortions are the same as in the unverifiable case with complete contracting.

Keywords: Asymmetric Information, Irreversibility, Long-term Contracts, Renegotiation, Uncertainty.

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1 Introduction

When a new durable good must be produced, for instance a network for water supply, the decision on its capacity is based on forecasts of future demand for this network. However, forecasting future demand implies inaccuracy for several reasons.

In the case of the construction of a water network, a public authority or a contractor might decide to build or improve the network in one area. While this work is being carried out, the public authority might realize that the forecast underestimated the amount of water passing through the network or that further improvement or enlargements are needed for some sections of the network. In many cases, if it is decided that additional work is necessary, the first part of the network can already be used before the additional work is finished and will thus generate a surplus. This surplus will be realized even though the entire project is not finished and the final (total) surplus is not yet enjoyed by the public. The same argument would hold for other types of infrastructure and networks, for instance another type of transportation network. One could also think of building a house of public utility (for instance a community centre, a hospital or a prison). If the local counsel decides that they will need to extend the building because the initial one is too small, the first, and hopefully finished, part of the building can often be used even though the second part is not yet operational.

Let us focus for a short while on transportation networks, and more precisely on road infrastructure. A clause that is common in contracts stipulate that the parties should share the financial consequences of unforeseen events in an equitable way. An example is the forecasting demand for a durable good for the next 20 or 30 years. Take the example of the Norwegian Public Roads Administration. When considering the National Transportation Plan for 2002-2011, the Norwegian Parliament (Stortinget) decided to implement three road projects as a pilot project for evaluating Public Private Partnerships in Norwegian road infrastructure. These three Public-Private Partnership contracts have specific clauses for who is responsible in different unforeseen situations. For instance, the Norwegian Public Roads Administration is responsible in the case of changes in the scope of service specifications during the operational phase and if changes in traffic occur[1].

Another situation where demand is subject to ex ante uncertainty is when a firm hires a software developer to create a customized software, the firm needs to decide upon the specifications of the software before it is created and without knowing exactly how it will fit into the everyday

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[1] For more information see www.vegvesenet.no

tasks of its employees. When the software is ready and the employees have been trained in using it, the firm might realize that further details can be added to improve its performance. If these improvements are important enough, the firm might ask the developer to further improve the already existing software. While this second version is being produced, the firm can still enjoy the benefits of the initial version. This article explores the idea that, because of ex ante uncertainty, initial decisions can be improved. But while waiting for the improvement to be finalized, the initial product already generates a surplus. This can be exploited by the firm in its decision to invest.

In this article, I study a setup where the initial gain from an irreversible project is subject to some uncertainty. Over time this uncertainty will disappear and the true value of the project will become known. The legal entity taking the investment decision (hereafter denominated ”the principal”) can exploit the fact that a second-period investment can be added to the initial investment and thus he can avoid unnecessary high costs in the first period. In this framework, there is a trade-off between investing under uncertainty (and obtaining the total surplus earlier) and waiting until the true value of the project is observed (and avoiding unnecessary spending on an irreversible investment). I study this problem in a framework where the principal not only faces a certain degree of uncertainty related to the value of his project (although he knows it is worth undertaking the investment), but he also faces agents with private information on their ability or cost of performing the desired task. In this setting the optimal first-period investment is positive but lower than when all the investment has to be done in the initial stage of the contractual relationship. This is due to the trade-off between investing under uncertainty and gaining (parts of) the first-period surplus on the one hand, and, on the other hand, delaying production until more information is available. Because of the asymmetric information between the principal and the agent, the optimal (“full information”) investment level is not always attainable. I derive the optimal investment level and the transfers allowing its implementation in an environment with asymmetric information. I show that this “asymmetric information” investment decision can be implemented when the future surplus is verifiable as well as when it is non verifiable. In the case of non verifiability I show how a contract can be implemented both under complete and incomplete contracting. Further, I compare the outcome and investment levels in the different settings. When uncertainty is unverifiable, I show that the distortion is stronger under complete contracting than under incomplete contracting. When uncertainty is verifiable, distortions are the same as in the unverifiable case with complete contracting.

This article is related to the (real) option value literature in that an investment is irreversible and the investors can get more information about the cost or value of a project by postponing the investment. McDonald and Siegel (1986) argue that a decision to invest does not only depend on whether at that point in time the project is profitable. It also depends on whether it would be more profitable or not to wait and learn more about the (uncertain) value of the project. As pointed out in Bernanke (1983), when agents have to take irreversible investment decisions, they have to consider the trade off between the extra return from an early investment and the benefit from learning more information when postponing the investment. Furthermore, Cukierman (1980) shows that when uncertainty increases, this trade-off implies that the gain from delaying an investment increases. A presentation of this literature can be found in Dixit and Pindyck (1994). The current article is different in that an early investment generates a surplus in the early stage but this investment can be completed by an additional investment once more information on the benefit of the project becomes available. Thus the surplus can be adjusted upward once there is more information available.

Contract theory has studied optimal contracts under asymmetric information for several decades. Baron and Myerson (1982) derived the optimal regulation schema for regulating a monopolist with private information about its cost parameter. Baron and Besanko (1984) showed that the optimal two-period contract under asymmetric information when the firm has the same cost parameter in both periods and the regulator can commit to what it will do in subsequent periods is the repetition of the initial one-period contracts since any modification due to new information would modify the behaviour in the first period and the second best would no longer be attainable. Laffont and Tirole (1986, 1987, 1988 and 1990) studied the optimal incentive contract in both single and multi-period models with no commitment as well as commitment and renegotiation. Lewis and Sappington (1989) derived condition for the optimal contract (and constraints) when the principal his unable to commit to more than one-period contracts. Other papers have studied the implication of renegotiation under asymmetric information. Dewatripont and Maskin (1990) and Laffont and Martimort (2002, chapter 9) gives a good overview of this literature. This paper uses a two-period contract theory model similar to those used in the papers mentioned above. The main difference is my assumption of irreversible investment.

All of these papers consider setups where the future is well defined or situations where future values can be approximated by the ex ante estimated value. In this paper I take a different approach and explore what would be the outcome if the knowledge about the surplus from an investment project changes over time. Put differently, I explore what would happen if, ex ante, only the probability distribution of the surplus is known, but once the true surplus is known the principal can adjust his decision to “fit” the actual surplus function.

Dewatripont and Maskin (1995) analyze a model with ex ante uncertainty about the extent to which the manager can reduce his own effort by investing in capital and hiring workers. Their analysis suggests that when renegotiation is possible, restricting ex ante what the owner can observe might be optimal and that their explanation can help explain why we often observe relatively simple contracts, where the only concern might be renegotiation.

The current article is also related to the literature on Public-Private Partnerships. In the introduction and the examples given in this article, I focus mostly on situations where the principal is a public authority and wants to contract out an investment project. In addition, in this paper, if the investment is made in the early stage there is still a cost to be paid in the second stage. One could easily imagine that this second-period cost could be some sort of maintenance or service cost. However, this article does not consider whether this is the optimal structure for delegating the project to the private sector. This is studied by Hart (2003) and Bennet and Iossa (2006) in an incomplete contracting environment and by Martimort and Pouyet (2008) in an complete contracting setting[2]. Iossa and Martimort (2008) provides a good overview of the PPP issue.

A recent part of the literature has focused on situations where the uncertainty disappears over time. In the first part of the situation studied here there is uncertainty, but in the second part the uncertainty disappears. Courty and Li (2000) analyze pricing schemes, for instance, for plane tickets, where the consumer buys the ticket before knowing the exact valuation for the good. However the consumer can, by choosing the "right" contract, get a refund in the second period if his valuation is lower than the price he paid in the first period. In this way, the monopoly can increase its profit. In my model, by investing at an early stage, before the exact surplus is known, the principal can increase his gain. In the second period, when the exact surplus is known, the principal can adjust his first-period investment to maximize surplus. Pavan, Segal and Toikka (2008) study incentive-compatible mechanisms in a dynamic environment where decisions can be made over time. However they focus on situations where agent's private information evolve over time and do not consider situations where the principal's information, and thus the public information, evolves over time.

This paper is organized as follows. Section 2 presents the model. In Section 3, I analyze the outcome when the principal can fully commit to what he will do for each realized outcome in the second period and the uncertainty variable is verifiable in the second period. In Section 4, extends the model to situations where the principal cannot contract upon the uncertainty variable ex ante because it is unverifiable. I consider both the case of incomplete and complete contracting under non-verifiability of the uncertainty variable. Section 5 compares the output of the optimal mechanisms derived in earlier sections and derives policy implications from these results. Finally Section 6 briefly summarizes and concludes my findings. Proofs are relegated to the Appendix.

2 The Model

I consider a two-period model in which a principal wants to delegate a task to an agent with private information on his productivity. The realization of this task creates a surplus for the economy (i.e. for consumers).

However, ex ante the surplus generated by this production is not known. The surplus in one period is thus denoted $S(q-\varepsilon)$, where q is the quantity produced and ε represents the ex ante

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[2] Although they also have a section on incomplete contracting.

uncertainty on consumer surplus. $S(.)$ is assumed to be strictly increasing and concave.

It is common knowledge that ε is distributed on $[\underline{\varepsilon}, \bar{\varepsilon}]$ according to the density function $f(\varepsilon)$. The associated cumulative distribution function is denoted $F(\varepsilon)$. After the first period the true value of ε is observed by both the principal and the agent. I also assume that the investment realized in the first period already generates a surplus in this period as well as in the second period.[3]

The agent's utility is given by the following function:

$$U(\theta) = t - \theta q$$

where t is the transfer received for the "production", q is the quantity produced and θ is the agent's productivity parameter. The latter is only observed by the agent.

I also assume that the agent has no outside opportunity and, thus, its reservation utility is equal to zero. However the agent does not need to break even in each period. He will accept a contract as long as his inter-temporal utility is non-negative.

The principal only knows that θ can take two values: $\underline{\theta}$ with probability ν and $\bar{\theta}$ with probability $1-\nu$, where $\bar{\theta} > \underline{\theta} > 0$ (and this is common knowledge). Type $\underline{\theta}$ can be seen as the "efficient" type and type $\bar{\theta}$ as the "inefficient" type. Furthermore, θ does not change over time (i.e. in this framework the agent cannot invest in order to increase his efficiency over time).

Denote β the weight of the first period in each player's utility function.

For given values of θ, ε and the investment in each period (denoted q_1 and q_2 respectively), the principal's surplus over the two periods is

$$\beta\left[S(q_1 - \varepsilon)\right] + (1-\beta)\left[S(q_1 + q_2 - \varepsilon)\right] - t$$

where $t = \beta t_1 + (1-\beta) t_2$[4].

Some comments about this surplus function are in order. The principal maximizes only the surplus that the production generates and ignores the value of any private rent enjoyed by the agent. I furthermore assume that the cost of producing q_i, $i = 1, 2$, occurs not only in the period when it is originally produced but also in the following period (if there is one). One could imagine that the produced good or service comes with some sort of maintenance cost. The reader should also notice that there is no depreciation of the first-period production between the periods and that the two productions can be added together without any additional cost.

Since ε is not known ex ante, the appropriate expression to consider is the expected value of the

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[3] For technical reasons, I also assume that the surplus is non negative and that $\forall\varepsilon$, $S'(-\varepsilon) > \bar{\theta}$.

[4] As will become clearer when solving the model, in most cases whether the transfer is made in period 1 or 2 is unimportant, only the total transfer matters. Therefore I am only interested in t. However, any transfer contingent on ε is necessarily given in the second period.

principal's surplus. That is

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big[\beta\left[S(q_1-\varepsilon)-t_1\right]+(1-\beta)\left[S(q_1+q_2-\varepsilon)-t_2\right]f(\varepsilon)d\varepsilon$$

Note that when no second period adjustment can be made and under complete information the optimal investment, q^*, is such that

$$E\left[S'(q^*(\theta)-\varepsilon)\right]=\theta \tag{1}$$

where $E[.]$ represents the expectation operator with respect to ε[5].

In the rest of this paper, and without loss of generality, I will restrict my attention to direct revelation mechanisms (with respect to the agent's cost).

In the current paper I only consider that one firm is needed in both periods. The reader might wonder if this is indeed optimal. In this model there are no externalities between the two periods/costs, so ex ante it does not matter if the principal uses the same firm in both periods and for both investments or not. This is because the probability distribution of cost parameters would be the same for both firms. A less technical argument would be to say that the principal and the agent are in a "locked-in" relationship and are thus committed to work together.

3 Full Commitment and Verifiability

In this section I will restrict my attention to the case where the principal is able to perfectly commit ex ante to a production- and transfer scheme for both periods and ε is assumed to be verifiable. The principal's offer to the agent will include a quantity q_1 to be produced in the first period. The contract will also include a second-period quantity. This quantity can be either positive or equal to zero. In the latter case, the contract is ex ante a one-shot investment contract. This second-period quantity will be produced after the realization of ε and since it is assumed that ε is verifiable, i.e. any contract that is contingent on the value of ε can be enforced by a court of law, it can depend on ε and is thus denoted $q_2(\varepsilon)$.

First I will analyze the case where the agent's type is common knowledge, i.e. there is no private information. Then I will turn to the case where only the agent has information about his type.

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[5] This is a standard maximization problem with only the firm's participation constraint and is left to the reader as an exercise.
Note also that if I allow for reversible investments (i.e. I allow $q_2(\varepsilon)$ to take negative as well as positive values) and there is no "cost of destruction", the first-period investment level would be the same as for a once-and-for-all investment.

Full Commitment and Symmetric Information

If θ is observed the principal's problem is

$$\max_{\{q_1(\theta),q_2(\varepsilon,\theta),t(\varepsilon,\theta)\}} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left[\beta S(q_1(\theta)-\varepsilon)+(1-\beta)S(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon)-t(\varepsilon,\theta)\right] f(\varepsilon)d\varepsilon \quad (2)$$

$$\text{subject to } E_\varepsilon[U(\varepsilon,\theta)] = E_\varepsilon[t(\varepsilon,\theta)-\theta q_1(\theta)-(1-\beta)\theta q_2(\varepsilon,\theta)] \geq 0 \quad (3)$$

$$q_2(\varepsilon,\theta) \geq 0 \quad (4)$$

The problem involves two decisions: the first-period investment and, after having observed the realization of ε, the most appropriate second-period investment given this value of ε. The second-period decision will obviously depend on ε, but it will also depend on the principal's decision to invest in the first period. In other words, the optimal second-period investment will depend on the first-period outcome as well as the realization of ε. Hence, I solve the principal's problem by backward induction. I start by looking at the second period and I then use the outcome of this to find the first-period optimal investment.

In the second period both q_1 and ε are observed and the principal's maximization problem is

$$\max_{q_2(\varepsilon,\theta)\geq 0} S(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon)-\theta q_2(\varepsilon,\theta) \quad (5)$$

The solution to this problem will depend on the realization of q_1 and ε and I obtain

$$S'\big(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon\big) = \theta \text{ if } S'(q_1(\theta)-\varepsilon) \geq \theta \quad (6)$$

$$q_2(\varepsilon,\theta) = 0 \text{ otherwise} \quad (7)$$

In words, if the first-period investment does not exceed the optimal investment level there will be a second-period investment to achieve the optimal investment level. However, if the first-period investment, q_1, exceeds the optimal investment for the realized ε there will be no second-period investment since it would be optimal to disinvest rather than invest more and this is not possible.

In this setting, having observed the realized value of the surplus function, the principal has the possibility to increase his investment to satisfy demand. In the software example presented in the introduction, this could be seen as a situation where the initial version of the software can be further improved to fully satisfy the needs of the firm.

Define $\phi(.)$ as being the inverse of $S'(.)$. Since $S(.)$ is concave this function is wel-defined. Using this notation, $q_2(\theta,\varepsilon)$ can be written more compactly as $q_2(\varepsilon,\theta) = max\{\phi(\theta)-q_1(\theta)+\varepsilon, 0\}$.

I will now consider what the principal's best strategy in the first stage of the relationship will be and use the information obtained from solving the second stage of the problem as a function of q_1 and ε.

Knowing what the second period investment is as a function of q_1 and that the agent's participation constraint will bind such that $E_\varepsilon[t(\varepsilon,\theta)] = \theta q_1(\theta) + (1-\beta)\theta E_\varepsilon\big[q_2(\varepsilon,\theta)\big]$ the principal's first-period problem when θ is known can be written

$$\max_{q_1(\theta)} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big[\beta S(q_1(\theta)-\varepsilon)+(1-\beta)S\big(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon\big)-\theta\big[q_1(\theta)+(1-\beta)q_2(\varepsilon,\theta)\big]\Big] f(\varepsilon)d\varepsilon. \tag{8}$$

Proposition 1. The optimal first period investment, $q_1(\theta)$, is characterized by

$$E\big[S'(q_1(\theta)-\varepsilon)\big]-\theta = (1-\beta)\int_{q_1(\theta)-\phi(\theta)}^{\bar{\varepsilon}} \big[S'(q_1(\theta)-\varepsilon)-\theta\big] f(\varepsilon)d\varepsilon. \tag{9}$$

It can be shown that the optimal first-period investment when there is a possibility of adjustment in the second period is lower than in the case with no possibility of adjusting the investment to the true surplus. Comparing the investment level given in Proposition 1 to the the investment level when no additional investment can be made, it is easy to see that the first-period investment is lower when there is possibility of adjustment. This is fairly intuitive. The principal has the possibility to delay parts of the (or the entire) irreversible investment until ε is known and does not have to make unnecessary costly investment (in the case of a low ε). But delaying production/investment to the second period has a cost, namely a lower first-period surplus. There is thus a trade-off between being prudent and waiting until the true value of ε is observed, and assuring that the surplus in the first period is as high as possible. In a more standard framework, the principal has no possibility of playing prudent since he cannot adjust his decision once ε is observed and thus in such a model the first-period investment will be higher.

Proposition 1 links this paper to the literature on the Precautionary Principle (Gollier, Jullien and Treich (1994) and Gollier and Treich (2000)). If the quantities in this article are interpreted as the level of investment against a certain type of undesirability (for instance filters against CO_2 emissions). Ex ante, there is uncertainty around how bad CO_2 emissions are for society. However, it is still optimal to start investing (in prevention against this undesirability) in period 1. Furthermore, as in Arrow and Fisher (1974) and Henry (1974), I obtain that the first-period investment is lowered so as to allow for more flexibility in period 2.

Full Commitment and Asymmetric Information

In the previous section, I analyzed the model in a setting with perfect information, I will now turn to the case where the agent's productivity parameter θ is private information and characterize the optimal contract under asymmetric information. The analysis will be similar to the previous one but with an additional "obstacle": The agent might want to hide his true marginal cost from the principal in order to obtain a higher profit from the contract.

Assuming that the principal is able to fully commit to what he will do in each of the two periods, he will offer a menu of contracts $\left\{ \big(\underline{q}_1, \underline{q}_2(\varepsilon), \underline{t}(\varepsilon)\big), \big(\bar{q}_1, \bar{q}_2(\varepsilon), \bar{t}(\varepsilon)\big) \right\}$ to the agent. In other words, I assume that ε is verifiable and can be contracted upon ex ante.

Whatever the marginal cost for the agent, he will accept the contract if and only if he obtains a non-negative utility from accepting it. However the principal will want to prevent the agent from obtaining more when he lies about his cost parameter than when he tells the truth. So, to prevent any type of agent to choose the contract designed for another type of agent, the menu of contracts proposed by the principal need also be incentive compatible. It is shown in the appendix that two of these four constraints can be neglected as long as the monotonicity condition $\underline{q}_1 + (1-\beta)E[\underline{q}_2(\varepsilon)] \geq \bar{q}_1 + (1-\beta)E[\bar{q}_2(\varepsilon)]$ is satisfied (this should be checked ex post). The principal's maximization problem can thus be written as follows.

$$\max_{(\underline{q}_1, \underline{q}_2, \underline{t}), (\bar{q}_1, \bar{q}_2, \bar{t})} \nu \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big[\beta S(\underline{q}_1 - \varepsilon) + (1-\beta) S\big(\underline{q}_1 + \underline{q}_2(\varepsilon) - \varepsilon\big) - \underline{t}(\varepsilon)\Big] f(\varepsilon) d\varepsilon +$$

$$+ (1-\nu) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big[\beta S(\bar{q}_1 - \varepsilon) + (1-\beta) S\big(\bar{q}_1 + \bar{q}_2(\varepsilon) - \varepsilon\big) - \bar{t}(\varepsilon)\Big] f(\varepsilon) d\varepsilon \qquad (10)$$

subject to

$$E\big[\bar{t}(\varepsilon)\big] - \bar{\theta}\big(\bar{q}_1 + (1-\beta)E\big[\bar{q}_2(\varepsilon)\big]\big) \geq 0 \qquad (11)$$

$$E\big[\underline{t}(\varepsilon)\big] - \underline{\theta}\big(\underline{q}_1 + (1-\beta)E\big[\underline{q}_2(\varepsilon)\big]\big) \geq E\big[\bar{t}(\varepsilon)\big] - \underline{\theta}\big(\bar{q}_1 + (1-\beta)E\big[\bar{q}_2(\varepsilon)\big]\big) \qquad (12)$$

Where equation 11 is the participation constraint of type $\bar{\theta}$ and equation 12 is the incentive constraint associated with type $\underline{\theta}$. The solution to this problem is characterized in the following Proposition.

Proposition 2. The optimal menu of contracts under adverse selection is characterized by:

1. The contract chosen by the efficient type allows for the optimal quantities to be invested but the agent is given a rent to induce him to choose this contract rather than the one designed for the inefficient type. Formally,

   - $\underline{q}_1$ is such that

   $$E\big[S'(\underline{q}_1 - \varepsilon)\big] - \underline{\theta} = (1-\beta) \int_{\underline{q}_1 - \phi(\underline{\theta})}^{\bar{\varepsilon}} \big[S'(\underline{q}_1 - \varepsilon) - \underline{\theta}\big] f(\varepsilon) d\varepsilon$$

   - $\underline{q}_2(\varepsilon) = \max\big\{\phi(\underline{\theta}) - \underline{q}_1 + \varepsilon, 0\big\}$.

   - $E\big[\underline{t}(\varepsilon)\big] = \underline{\theta}\big(\underline{q}_1 + (1-\beta)E\big[\underline{q}_2(\varepsilon)\big]\big) + \Delta\theta\big(\bar{q}_1 + (1-\beta)E\big[\bar{q}_2(\varepsilon)\big]\big)$

2. The contract chosen by the inefficient type includes downwardly distorted outputs and no rent.

- $\bar{q}_1$ is such that

$$E\left[S'(\bar{q}_1-\varepsilon)\right]-\bar{\theta}-\frac{\nu}{1-\nu}\Delta\theta=(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}^{\bar{\varepsilon}}\left[S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{\nu}{1-\nu}\Delta\theta\right]f(\varepsilon)d\varepsilon$$

- $\bar{q}_2(\varepsilon)=\max\{\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)-\bar{q}_1+\varepsilon,0\}$.
- $E\left[\bar{t}(\varepsilon)\right]=\bar{\theta}\left(\bar{q}_1+(1-\beta)E\left[\bar{q}_2(\varepsilon)\right]\right)$

Proposition 2 shows that the investment (in both periods) of the efficient type is equal to the optimal level described in the full-commitment and perfect-information case. To induce this agent to choose "the contract designed for him", he receives a positive rent which is equal to the difference in productivity of the two types times the total quantity demanded of the inefficient type. Moreover, the investment of the inefficient type is distorted downwards and this agent enjoys no rent.

The result is very similar to the standard result in principal-agent models with adverse selection. In Proposition 2 the cost of investing in period one also takes into account the possibility of adjusting upward the level of investment in the second period. This additional cost associated with an exaggerated irreversible investment (compared to the basic model) is represented by the expression on the right hand side of the equations characterizing first-period quantities and it can be interpreted as the opportunity cost of an excessively high, and thus unnecessary, first-period quantity. It is this term that makes the first-period investment lower than in the case with no possibility for second-period adjustments. Furthermore, the first-period investment is also lower than in the case of uncertainty and no private information. Thus there are two forces driving down the investment level; first the uncertainty and possible of readjustment in the second period pulls down the level of first-period investment. This is because the gain from being slightly prudent and not make excessively high irreversible investments under uncertainty is higher than the potential gain from this investment in the first-period. Second, the presence of asymmetric information further distorts the level of investment of the inefficient type. These two distortions go in the same direction and the first-period investment of the inefficient type is reduced. It can also be shown[6] that the total level of investment under asymmetric information is lower than in the case with uncertainty and no private information. The possibility of adjusting the initial investment does not entirely compensate the information asymmetry.

Note that I have assumed that the agent will choose to participate as long as he enjoys a non negative intertemporal transfer. He does not necessarily need to break even in each period. Due to this assumption I only know the value of the intertemporal transfer but cannot be more precise regarding what the agent will get in each period. In any case, the transfer variable t can be interpreted as the discounted value of the total transfer. A simple way of structuring the

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[6] See Proposition 5 for more details on this.

payment to the agents would be to give to each agent who accepts the contracts a first-period transfer that covers exactly his cost, and nothing else, and any rent could then be included in the second-period transfer.

To obtain the previous Proposition I have assumed that ε is observable by a third party and thus any contract contingent on the realized value of ε can be enforced by a court of law. However, not everything can always be observed by outside parties. In the next Section I will explore what happens if the realization of ε cannot be used when writing the initial contract. In the case of complete contracting, the outcome in Proposition 2 is still obtainable. But under incomplete contracting total output will be higher[7] and surplus lower compared to the results in Proposition 2.

4 Unverifiability

In this section I will assume that ε, and thus the surplus generated by the project, is observable by all parties, but it is unverifiable information. In other words, even though all parties observe the realization of ε this information will not be valid in front of a court of law. I will continue assuming that the principal can commit to a long-term contract, but now the long-term contract cannot depend directly on the true realization of ε.

For instance, the contract for improving the network cannot specify what what will be done if the traffic survey turns out to be wrong. This will have to be dealt with at the time when the partners realize that the first-period contract is not the optimal contract.

I will consider two different approaches to this problem. First I will take an "incomplete-contract" approach. That is, the initial contract will not stipulate any second-period quantity/transfer. But just before the second period, when having observed ε, the principal can offer an additional contract to "fit" the realization of the uncertainty parameter. This would be the most natural way to handle situations concerning characteristics of the service itself. These would be events that occur over time but that cannot be anticipated at the time of contracting (but once they occur they become observable to both parties without any cost). This could for instance be the need for additional rooms for computers in schools once internet is created or hospitals needing different kinds of warden or operating room following new clinical innovations. These characteristics capture changes in users' needs that have an impact on the characteristics of the final product. In PPPs this is also related to the "change mechanism", a clause in the contract which, acknowledging the possibility that investment needs changes in the future but being unable at the time to anticipate the change, describes the way the parties will have to negotiate to complete the contract at a later contractual stage. Finally, it is also a good example of what might happen in practice with "add ons" where the original contract is

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[7] Higher for the inefficient type and equal for the efficient type

completed over time.

Secondly, I will analyze this model in a “complete-contract” setting. In other words, when the initial contract stipulates the second-period quantity and transfer as a function of the efficiency of the agent (θ) as well as how the parties will behave once they observe the uncertainty variable. A good example for this would be traffic forecasts. The parties sign a contract that stipulates the first-period investment given what information is available ex ante (i.e. the traffic forecast). But they keep in mind that this forecast might be wrong and therefore specify in the initial contract how they will react if the realized traffic volume is different from the forecasted volume.[8]

Incomplete Contracting

In this section I will analyze the situation where the initial contract does not stipulate any new investment in the second period[9]. But just before the second period, when ε has been realized and observed, the principal can choose to add an extra output-transfer pair (q_2, t_2) to the contract. Typically, the principal would like to increase the investment if the realization of ε shows that the initial investment was suboptimal.

The timing of the contracting can now be described as follows:

- θ is realized and observed by the agent.
- The principal offers a menu of two-period contracts $\left\{(\underline{q}_1, \underline{t}), (\bar{q}_1, \bar{t})\right\}$ where q_1 is provided in period one, but, as explained previously, also has a cost in the second period.
- The agent accepts or refuses the contract.
  If he accepts, he announces his θ.
  If he rejects, the game ends and both players get a pay-off equal to zero.
- q_1 is realized.
- ε is observed by all players.
- The principal has the possibility to offer a second-period contract[10].

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[8] I am grateful to Elisabetta Iossa for her suggestions for motivating the two approaches I consider.

[9] Note that I have not considered that the initial contract can include a second-period investment level and transfer that is independent of ε. In fact it can easily be shown that such investment and transfers are not optimal and should be set equal to 0. At the renegotiation stage, this would increase the rent given to the agent to make him accept the second-period investment. Ex ante, if such output and transfers are included in the initial contract, the principal could do better by including them in the first-period investment. This would generate the same rent to the agent, but the principal would benefit from the additional level of investment (regardless of whether the total first-period investment is *too high* or not) and not just throw away money. I.e to ex ante impose a fixed (q_2, t_2) only results in higher transfers because of rent issues.

[10] Although the initial contract is independent of ε, the surplus function is known from the start.

- The agent accepts or refuses the additional contract.
  If he accepts, q_1 is supplemented by q_2.
  If he refuses, the two parts are bound by the initial contract.
- If necessary, q_2 is realized.

An important assumption for my analysis is that the principal wants to include both types of agents in the revised contract. Otherwise he would only offer a renegotiated contract containing the first-best level of add-on provision and a transfer equal to the cost of providing this for $\underline{\theta}$. Only $\underline{\theta}$ would accept such a contract.

The form of the revised contract might depend on who obtains information on ε before the second period. I will assume that in period two, ε is common knowledge.

I assume that both the principal and the agent observe the realization of ε between the two periods and they do not incur any cost to observe this realization, but it is the principal who decides to renegotiate the initial contract. In the infrastructure example given in the introduction, one could imagine that the estimation of the demand for water network is carried out by an independent institution. And even in the case where the demand is estimated by the public authority, the (approximate) level of demand cannot be regarded as private information. Assume for instance that a new residential area is being created. This will affect the demand for the water infrastructure but the public authority has no means to keep this development a secret since everyone will know about the plans to build new homes and will be able to infer the effect this has on the demand for water infrastructure. Furthermore, in the infrastructure example, it is fairly easy to imagine that if the public authority (i.e. the principal) realizes that the initial investment is not enough, he will approach the building company to add necessary adjustments to the initial contract. If the reason for renegotiation was related to the production technology[11], one could also assume that renegotiation was proposed by the building company (i.e. the agent). Here I focus on situations where a possible insufficiency of the initial contract is due to demand itself, not to the production technology. I therefore focus on renegotiation initiated by the principal.

My analysis will be carried out in three steps. First I will study the optimal additional contract following “good” news. That is, when the agent has announced $\underline{\theta}$ in the initial contract. Then I will study the additional contract following “bad” news. That is, when the agent has announced $\bar{\theta}$ in the initial contract. Finally, I will turn to the optimal initial contract when the principal can commit over two periods but the contract cannot depend on ε[12].

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[11] For instance, if the terrain needs more preparation or material to make the network safe.

[12] The approach that I use is that of Laffont and Tirole (1990).

The “Efficient” Branch

In the case where the agent has chosen to announce $\underline{\theta}$ in the first period, the principal knows that he is dealing with the efficient type since only this type would get a non-negative utility from choosing this contract. Thus it is optimal to choose $\underline{q}_2(\underline{\theta})$[13] such that $S'(\underline{q}_1 + \underline{q}_2(\varepsilon, \underline{\theta}) - \varepsilon) = \underline{\theta}$ (if further investment is desirable).

For the agent of type $\underline{\theta}$ to be indifferent between choosing the bunching branch and choosing the “efficient” branch, he needs to enjoy the same level of rent regardless of which branch he chooses. So the transfer in the case where the agent chooses to truthfully announce that he is of type $\underline{\theta}$ will be set so as to make the agent indifferent between the two total transfers.

The Bunching Branch

Following the outcome of the first period, the principal updates his beliefs about the agent's type. Denote ν_2 the updated probability that the agent is of type $\underline{\theta}$ when he has chosen the bunching contract. $\nu_2 = \frac{(1-x)\nu}{(1-x)\nu+1-\nu}$ where x is the probability that the efficient type announces his true type in the first period.

The optimization problem for the additional contract is

$$\max_{(\underline{q}_2(.),\underline{t}_2(.)),(\bar{q}_2(.),\bar{t}_2(.))} \nu_2\Big(S(\bar{q}_1+\underline{q}_2(\varepsilon,\bar{\theta})-\varepsilon)-\underline{t}_2(\varepsilon,\bar{\theta})\Big)+(1-\nu_2)\Big(S(\bar{q}_1+\bar{q}_2(\varepsilon,\bar{\theta})-\varepsilon)-\bar{t}_2(\varepsilon,\bar{\theta})\Big) \tag{13}$$

subject to the two types participation and incentive constraints as well as $\bar{q}_2(\varepsilon,\bar{\theta}) \geq 0$, $\underline{q}_2(\varepsilon,\bar{\theta}) \geq 0$.

It can be checked that the two binding constraints are

$$\bar{t}_2(\varepsilon,\bar{\theta}) = \bar{\theta}\bar{q}_2(\varepsilon,\bar{\theta}) \tag{14}$$

$$\underline{t}_2(\varepsilon,\bar{\theta}) = \underline{\theta}\underline{q}_2(\varepsilon,\bar{\theta}) + \Delta\theta\bar{q}_2(\varepsilon,\bar{\theta}) \tag{15}$$

Looking at the first-order conditions for this problem gives the same result as previously. That is,

- If $S'(\bar{q}_1 - \varepsilon) < \bar{\theta} + \frac{\nu_2}{1-\nu_2}\Delta\theta$ (respectively $< \underline{\theta}$), for any given ε, then the additional contract will stipulate $\bar{q}_2(\varepsilon,\bar{\theta}) = 0$ (respectively $\underline{q}_2(\varepsilon,\bar{\theta}) = 0$).
- Otherwise $\underline{q}_2(\varepsilon,\bar{\theta})$ is such that $S'(\bar{q}_1 + \underline{q}_2(\varepsilon,\bar{\theta}) - \varepsilon) = \underline{\theta}$. The output of the efficient type is not distorted away from its first best level.

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[13] The θ in brackets indicates what type the agent claimed to be in the first period.

- Furthermore $\bar{q}_2(\varepsilon,\bar{\theta})$ is such that $S'(\bar{q}_1+\bar{q}_2(\varepsilon,\bar{\theta})-\varepsilon)=\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta$. The output of the inefficient type is distorted downward to decrease the efficient type's rent.

Characterizing the Initial Contract

Denote $\underline{T}(\varepsilon,\underline{\theta})=\underline{t}+(1-\beta)\underline{t}_2(\varepsilon,\underline{\theta})$, $\underline{T}(\varepsilon,\bar{\theta})=\bar{t}+(1-\beta)\underline{t}_2(\varepsilon,\bar{\theta})$ and $\bar{T}(\varepsilon,\bar{\theta})=\bar{t}+(1-\beta)\bar{t}_2(\varepsilon,\bar{\theta})$. For instance, if a $\underline{\theta}$-type chooses the bunching branch in the first period, his total transfer, as a function of ε, is denoted $\underline{T}(\varepsilon,\bar{\theta})$. Using these notations and knowing what will be the outcome of the renegotiation stage, the principal's maximization problem can be written

$$\begin{aligned}\max_{\left\{x,(\underline{q}_1,\underline{t})(\bar{q}_1,\bar{t})\right\}} \; & x\nu\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\left\{\beta S(\underline{q}_1-\varepsilon)+(1-\beta)S(\underline{q}_1+\max\{0,\phi(\underline{\theta})+\varepsilon-\underline{q}_1\}-\varepsilon)-\underline{T}(\varepsilon,\underline{\theta})\right\}f(\varepsilon)d\varepsilon \\ & +(1-x)\nu\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\left\{\beta S(\bar{q}_1-\varepsilon)+(1-\beta)S(\bar{q}_1+\max\{0,\phi(\underline{\theta})+\varepsilon-\bar{q}_1\}-\varepsilon)-\underline{T}(\varepsilon,\bar{\theta})\right\}f(\varepsilon)d\varepsilon \\ +(1-\nu)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} & \left\{\beta S(\bar{q}_1-\varepsilon)+(1-\beta)S(\bar{q}_1+\max\{0,\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)+\varepsilon-\bar{q}_1\}-\varepsilon)-\bar{T}(\varepsilon,\bar{\theta})\right\}f(\varepsilon)d\varepsilon \quad (16)\end{aligned}$$

Subject to

$$E\left[\underline{T}(\varepsilon,\underline{\theta})\right]-\underline{\theta}\left(\underline{q}_1+(1-\beta)E\left[\underline{q}_2(\varepsilon,\underline{\theta})\right]\right)\geq 0 \quad (17)$$

$$E\left[\bar{T}(\varepsilon,\bar{\theta})\right]-\bar{\theta}\left(\bar{q}_1+(1-\beta)E\left[\bar{q}_2(\varepsilon,\bar{\theta})\right]\right)\geq 0 \quad (18)$$

$$E\left[\underline{T}(\varepsilon,\underline{\theta})\right]-\underline{\theta}\left(\underline{q}_1+(1-\beta)E\left[\underline{q}_2(\varepsilon,\underline{\theta})\right]\right)\geq E\left[\bar{T}(\varepsilon,\bar{\theta})\right]-\underline{\theta}\left(\bar{q}_1+(1-\beta)E\left[\bar{q}_2(\varepsilon,\bar{\theta})\right]\right) \quad (19)$$

$$E\left[\bar{T}(\varepsilon,\bar{\theta})\right]-\bar{\theta}\left(\bar{q}_1+(1-\beta)E\left[\bar{q}_2(\varepsilon,\bar{\theta})\right]\right)\geq E\left[\underline{T}(\varepsilon,\underline{\theta})\right]-\bar{\theta}\left(\underline{q}_1+(1-\beta)E\left[\underline{q}_2(\varepsilon,\underline{\theta})\right]\right) \quad (20)$$

$$0\leq x\leq 1 \quad (21)$$

As usual constraints (18) and (19) are binding[14] and the problem can be rewritten with only the last constraint ($0\leq x\leq 1$). Denote $W(\underline{q}_1,\bar{q}_1,x)$ the objective function of the principal[15].

$\underline{q}_1$ and $\bar{q}_1$ do not enter the constraint and their optimal value (as a function of x) are[16]:

- The first-period quantity for the efficient type is at its first-best level. Formally $\underline{q}_1$ is such that

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\left\{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon=(1-\beta)\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\left\{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon. \quad (22)$$

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[14] I also ignore the participation and incentive constraint of a $\underline{\theta}$-type mimicking a $\bar{\theta}$-type in the first period since these constraints do not add any information with respect to the constraints already stated above. This is because, whatever a $\underline{\theta}$-type chooses to do in the first period, he still obtains the same rent.

[15] Its detailed expression can be found in the Appendix.

[16] Because the constraint only involves x, optimizing first with respect to the qs and then with respect to x is equivalent to optimizing jointly over the qs and x.

- However, the inefficient type's output is distorted $\bar{q}_1$ is such that

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \{S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{xv}{(1-x)v+(1-v)}\Delta\theta\}f(\varepsilon)d\varepsilon =$$

$$(1-\beta)\Bigg[\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{v_2}{1-v_2}\Delta\theta)}^{\bar{\varepsilon}} \{S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{xv}{(1-x)v+(1-v)}\Delta\theta\}f(\varepsilon)d\varepsilon$$

$$+\frac{(1-x)v}{(1-x)v+(1-v)}\int_{\bar{q}_1-\phi(\underline{\theta})}^{\bar{q}_1-\phi(\bar{\theta}+\frac{v_2}{1-v_2}\Delta\theta)} \{S'(\bar{q}_1-\varepsilon)-\underline{\theta}\}f(\varepsilon)d\varepsilon\Bigg]. \tag{23}$$

It is shown in the Appendix that if $\beta \in (0,1)$, then full bunching is never optimal. In other words, it is never optimal to have both types announce $\bar{\theta}$ in the first period and only separate them in the second period. Furthermore, for $(1-\beta)\phi(\bar{\theta})(\Delta\theta)^2$ small enough, full separation is optimal. If $(1-\beta)\phi(\bar{\theta})(\Delta\theta)^2$ is too big, some degree of bunching is optimal.

In the two extreme cases, $\beta = 1$ and $\beta = 0$, the model simplifies drastically and I get very obvious results.

In the case of $\beta = 1$ full separation is optimal. This is very intuitive, since the second period does not count in the objective function. Only the first-period gain counts, so we are back to a static adverse selection model.

When $\beta = 0$, $\frac{\partial W}{\partial x} = 0$, but $\forall \underline{q}_1, \bar{q}_1, W(\underline{q}_1, \bar{q}_1, x) \leq W(0,0,x)$. Thus, in this very particular case, no investment should take place in the first period. This is quite intuitive, since the first period does not count in the objective function. Only the second-period surplus counts and there is no gain from investing earlier (under uncertainty), thus we are again back to a static adverse selection model.

Finally, the following Proposition describes the optimal initial contract under asymmetric information.

Proposition 3. For $(1-\beta)\phi'(\bar{\theta})\Delta\theta$ small enough, under adverse selection, the contract yields no distortion at the top in both periods. But the inefficient type's output is distorted to avoid giving up too much rent in both the first and the second period.

1. The first-period quantity for the efficient type in the first period is at its first-best level and the agent enjoys a strictly positive rent. Formally $\underline{q}_1$ and the corresponding transfer are such that

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\}f(\varepsilon)d\varepsilon = (1-\beta)\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}} \{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\}f(\varepsilon)d\varepsilon \tag{24}$$

$$\underline{t} = \underline{\theta}\underline{q}_1 + \Delta\theta\bar{q}_1 + \Delta\theta E\left[\bar{q}_2(\varepsilon,\bar{\theta})\right] \tag{25}$$

2. However, the inefficient type's output is distorted. $\bar{q}_1$ is such that

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \{S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{x\nu}{(1-x)\nu+(1-\nu)}\Delta\theta\}f(\varepsilon)d\varepsilon =$$

$$(1-\beta)\Big[\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}} \{S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{x\nu}{(1-x)\nu+(1-\nu)}\Delta\theta\}f(\varepsilon)d\varepsilon$$

$$+\frac{(1-x)\nu}{(1-x)\nu+(1-\nu)}\int_{\bar{q}_1-\phi(\underline{\theta})}^{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)} \{S'(\bar{q}_1-\varepsilon)-\underline{\theta}\}f(\varepsilon)d\varepsilon\Big] \tag{26}$$

$$\bar{t}=\bar{\theta}\bar{q}_1 \tag{27}$$

3. In the second period, the efficient type always produces at the first-best level and enjoys a rent while the ineffcient type's output is less than the first-best level and this agent gets no rent from the contract. Formally

$$\underline{q}_2(\varepsilon,\underline{\theta})=\max\{0,\phi(\underline{\theta})+\varepsilon-\underline{q}_1\} \tag{28}$$

$$\bar{q}_2(\varepsilon,\bar{\theta})=\max\{0,\phi(\bar{\theta})+\varepsilon-\bar{q}_1\} \tag{29}$$

$$\underline{t}_2(\varepsilon,\underline{\theta})=\underline{\theta}\underline{q}_2(\varepsilon,\underline{\theta}) \tag{30}$$

$$\bar{t}_2(\varepsilon,\bar{\theta})=\bar{\theta}\bar{q}_2(\varepsilon,\bar{\theta}) \tag{31}$$

Compared to the optimal contract when ε is observable (but there is asymmetric information), an "incomplete" contract yields higher first-period and total level of investment for the inefficient type. Two effects force the principal to choose a contract that is less desirable than what he can reach when he can contract upon ε. First, the possibility for the efficient type to bunch with the inefficient type in the first period makes a downward distortion of the output less desirable. The distortion is there to decrease the rent the principal has to give up to make sure that the contracts are incentive compatible. The possibility of bunching weakens this need for incentive compatibility. Furthermore, the lack of ability to not behave opportunistically in the second period forces the principal to chose a different menu of contracts than under verifiability.

Complete Contracting

I will now characterize the initial contract in the complete-contracting framework with non-verifiablility of ε. In this setting the initial contract will specify the outcome of both periods and there will be no renegotiation between periods. The contract can not be contingent on ε but it can depend on the announcement the parties make of what they have observed between the two periods. In other words, q_2 will depend on what the agent and the principal claim to be the observed value of ε. The menu of contracts offered will be of the form

$\left\{\left(\underline{q}_1,\underline{q}_2(\varepsilon_a,\varepsilon_p),\underline{t},\underline{t}_2(\varepsilon_a,\varepsilon_p)\right),\left(\bar{q}_1,\bar{q}_2(\varepsilon_a,\varepsilon_p),\bar{t},\bar{t}_2(\varepsilon_a,\varepsilon_p)\right)\right\}$. The additional problem will be to induce the two contracting parties to announce the true ε.

This Section is based on the implementation literature (Maskin (1999) and Maskin and Moore (1999)) where the unverifiability problem can be circumvented by adequatly designed mechanisms ex ante.

One could imagine contracts with different rules for the announcements. However, here I will first focus on contracts where the two parts make their announcements simultaneously and then, for reasons that will become obvious, modify the rule so as to implement uniquely the desired outcome. For more details on contracting under nonverifiability, see for instance Laffont and Martimort (2002, chapter 6).

The timing of the contracting can now be described as follows:

- θ is realized and observed by the agent.
- The principal offers a menu of two-period contracts

  $$\left\{\left(\underline{q}_1,\underline{q}_2(\varepsilon_a,\varepsilon_p),\underline{t},\underline{t}_2(\varepsilon_a,\varepsilon_p)\right),\left(\bar{q}_1,\bar{q}_2(\varepsilon_a,\varepsilon_p),\bar{t},\bar{t}_2(\varepsilon_a,\varepsilon_p)\right)\right\}$$

  where q_1 is produced in period one, but needs maintenance in the second period, and q_2 is produced in period two according to the observed realization of ε announced by the principal and the agent.
- The agent accepts or refuses the contract.
  If he accepts, he announces his θ.
  If he rejects, the game ends.
- q_1 is realized.
- ε is observed
- The principal and the agent report what value of ε they have observed.
- q_1 is supplemented by q_2 in accordance with the parties announcements.

I will start this discussion by assuming that the principal and the agent reports *simultaneously* what value of ε they have observed. One way to implement this is to state that if the players disagree upon what ε has been realized the second-period investment and transfer will be equal to zero. However, if they announce the same ε, the quantity invested will be the one given by the formula for the optimal (second-best) investment under full commitment where ε is replaced by the announced $\varepsilon_a(=\varepsilon_p)$.

It can be shown that $\varepsilon_a = \varepsilon_p = \varepsilon$ is a Nash equilibrium in this "game". However, this outcome is not the unique equilibrium of this direct mechanism. Any announcement, ε', such that the two announcements coincide (and such that $S(q_1 + q_2(\varepsilon') + \varepsilon) > \theta(q_1 + q_2(\varepsilon')))$ is a Nash equilibrium in the second stage of the contract. To avoid this undesirable effect, in what follows I will look for a contract where there is no multiplicity of equilibria.

So in order to avoid the multiplicity of equilibria problem, the parties need to be "punished" when they lie. The best feasible outputs are still the ones described under full commitment. In the contract I will describe here, the first-period part (everything that does not depend on ε) stays the same as under full commitment[17], but the second-period part is modified. The realized output in period two will depend on the principal's announcement of ε, but there will be two transfers, one going from the principal to the agent, denoted t_p, and another one going from the agent to the principal, denoted t_a. It might seem strange that the agent has to pay the principal something when he is the one producing something, but the additional payments cancel at equilibrium and act in such a way that deviation from the equilibrium punishes the deviator.

Sub-game perfect Nash implementation (see Moore and Repullo (1988)) would be a further refinement. However, this requires that the players move sequentially. By changing the assumption that the principal and the agent announces *simultaneously* the value of ε that they have observed into a *sequential* game where the agent moves first[18]. The following transfers are the unique sub-game perfect Nash-equilibrium of the game and therefore ensure the implementation of the same asymmetric information outcome as under full commitment

Proposition 4. In the second period of the contract the following transfers imply that the unique outcome is the same as under full commitment.

$$\underline{t}_p(\varepsilon_a, \varepsilon_p) = S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_a) + \underline{\theta}\underline{q}_2(\varepsilon_p)$$

$$\underline{t}_a(\varepsilon_a, \varepsilon_p) = S(\underline{q}_1 + \underline{q}_2(\varepsilon_p) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_p)$$

$$\bar{t}_p(\varepsilon_a, \varepsilon_p) = S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a) + \bar{\theta}\bar{q}_2(\varepsilon_p)$$

$$\bar{t}_a(\varepsilon_a, \varepsilon_p) = S(\bar{q}_1 + \bar{q}_2(\varepsilon_p) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_p) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_p)$$

Notice first that when the two parties tell the truth, the above transfers reduce to the transfers paid under full commitment and verifiability. However, if one (or both) do not tell the truth,

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[17] Any expected rent from the second-period investment is included in the initial transfer.

[18] See Appendix for further details.

this party punishes itself. The only equilibrium outcome with these transfers is the outcome obtained when ε is verifiable.

5 Output Comparison

In previous sections I have derived the optimal mechanism and the optimal level of investment under different assumptions. I have only showed what would be the optimal outcome if the principal decide to contract out a project in the different environments and I have made very weak attempts to compare these outcomes. In this section I will compare the levels of investment under the different assumptions of verifiability of ε and type of contracting (i.e. incomplete versus complete contracting). In the presence of asymmetric information, it is clear from Proposition 4 that in the case with verifiability of ε and the case where ε is unverifiable but a complete contract can be written the investment levels are the same. The more interesting case is to compare the complete and incomplete contracting setting when ε is supposed to be observable in period two but unverifiable[19].

Note that the levels of investment for the efficient type are always at the same (symmetric information) level[20]. Thus the relevant investment levels to compare are the first- and second-period investments for the inefficient type, $\bar{q}_1$ and $\bar{q}_2(\varepsilon)$.

It can be checked that the levels of investment under incomplete contracting satisfy the participation and incentive constraints under complete contracting. Thus the levels of investment under incomplete contracting are feasible also under complete contracting. However, the solution to the complete contracting maximization problem shows that this is not the preferred (optimal) solution. In this way, complete contracting dominates incomplete contracting. Intuitively, complete contracting gives the contracting parties an extra *tool* (i.e. they can specify ex ante how to react ex post) and more instruments allows them to perform better.

However, even though the complete contracting outcome dominates the incomplete contracting outcome in the sense that it increases the principal's net surplus, it would be interesting to compare the total level of investment (first-period and second-period investment) realized under the two types of contract. In fact, it can be shown[21] that under complete contracting the total level of investment for the inefficient type is lower than under incomplete contracting. This result holds regardless of the value of ε. Thus, a fortiori, ex ante the inefficient type's level of investment under complete contracting is lower than under incomplete contracting. Furthermore,

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[19] Since the complete contracting and unverifiability case gives the same levels of investment as under verifiability, this can also be seen as a comparison of the latter case to the incomplete contracting and unverifiability setting.

[20] Except in the case of non verifiability and incomplete contracting where some efficient types might choose to bunch with the inefficient types in the first period and thus produce at a lower level.

[21] See proof of Proposition 5 in the Appendix.

both these levels of investments are lower than the symmetric information level of investment (ex ante and ex post).

Proposition 5. Both ex ante and ex post the inefficient type's total level of investment under complete contracting ($\bar{q}^c$), under incomplete contracting ($\bar{q}^i$) and his symmetric information level ($\bar{q}^{SI}$) can be ordered as follows

$$\bar{q}^c \leq \bar{q}^i \leq \bar{q}^{SI}. \tag{32}$$

In all cases except under bunching in incomplete contracting, the total level of investment for the efficient type is at its symmetric information optimal level.

Proposition 5 links the current paper to the literature on pre-commitment versus flexibility. In that strand of literature there is a trade-off between pre-committing and retaining flexibility on the output/investment decision until uncertainty has been resolved. Spencer and Brander (1992) study this trade-off in an oligopoly setting where there is ex ante uncertainty on demand. Appelbaum and Lim (1985) study the trade-off between pre-commitment versus flexibility on a firm's decision to deter entry. They show that in general a firm will make some pre-commitments in order to affect the probability of entry of other firms. In the current paper, pre-commitment can be viewed as being able to write a complete contract and flexibility as an incomplete contract. A complete contract has been shown to yield a higher net surplus to the principal than an incomplete contract. However the higher surplus comes at the cost of lower investment levels for the inefficient type (this is needed to keep the rent of the efficient type low). On the other hand, an incomplete contract yields higher investment levels for the inefficient type. But the cost of increasing this investment level is a decrease in net surplus to the principal. This is because separating the efficient type from the inefficient type now becomes more difficult.

6 Conclusion

Let me briefly recall the setting of this paper. I have studied a model where uncertainty disappears over time. In addition to the "global" uncertainty, the agent also has private information about his efficiency. The model has two periods, one with uncertainty and one without. Furthermore, any investment in the first period also yields a gain in the second period (but needs maintenance). The principal can take actions both under and after uncertainty to maximize his gain. I have shown that when the principal is able to commit to what he will do in forthcoming periods, regardless of whether the uncertainty parameter is verifiable or not, the first-period investment will be lower than when no additional investment can be implemented to fit the true surplus and asymmetric information further distorts the level of investment. I have also compared the levels of investment achieved under different types of contracting.

The framework studied in this article might help explain situation characterized by initial uncertainty and why, in such situations, contracts and production plans might be adjusted as time goes by and the agents (including the principal) learn more about the value of the task they perform.

In my model, the unobservability of the uncertainty variable before contracting is an assumption. Aline of future research is to try to endogenize the information structure. The firm can invest ex ante to gather more or less information about future demand.

Appendix

Proof of Proposition 1, 2 Lemmas used in the proof of Proposition 2, proof of Proposition 2-6 follow.

Proof of Proposition 1: The principal optimization problem is

$$\max_{q_1(\theta),t}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\Big[\beta S(q_1(\theta)-\varepsilon)+(1-\beta)S\big(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon\big)-t(\theta)]f(\varepsilon)d\varepsilon \tag{A1.1}$$

subject to

$$E_\varepsilon[U(\theta)]=E_\varepsilon[t(\theta)-\theta q_1(\theta)-(1-\beta)\theta q_2(\varepsilon,\theta)]\geq 0 \tag{A1.2}$$

where $q_2(\varepsilon,\theta)$ is given by the optimal second period contract.

The fact that (A1.2) will bind at the maximum and rearranging (A1.1) yield:

$$\max_{q_1(\theta)}\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\Big[\beta S(q_1(\theta)-\varepsilon)+(1-\beta)S\big(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon\big)-\theta\big[q_1(\theta)-(1-\beta)q_2(\varepsilon,\theta)\big)\Big]f(\varepsilon)d\varepsilon \tag{A1.3}$$

$$\Leftrightarrow \max_{q_1(\theta)}\beta\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}S(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon-\theta q_1(\theta)+(1-\beta)\int_{\underline{\varepsilon}}^{\varepsilon(\theta)^*}S(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon$$
$$+(1-\beta)\int_{\varepsilon(\theta)^*}^{\bar{\varepsilon}}\big\{S\big(q_1(\theta)+q_2(\varepsilon,\theta)-\varepsilon\big)-\theta q_2(\varepsilon,\theta)\big\}f(\varepsilon)d\varepsilon \tag{A1.4}$$

where $\varepsilon(\theta)^*=q_1(\theta)-\phi(\theta)$

Substituting $q_2(\varepsilon,\theta)$ by its expression yields

$$\max_{q_1(\theta)}\beta\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}S(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon-\theta q_1(\theta)+(1-\beta)\int_{\underline{\varepsilon}}^{\varepsilon(\theta)^*}S(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon$$
$$+(1-\beta)\int_{\varepsilon(\theta)^*}^{\bar{\varepsilon}}\big\{S\big(\phi(\theta)\big)-\theta\big(\phi(\theta)-q_1(\theta)+\varepsilon\big)\big\}f(\varepsilon)d\varepsilon \tag{A1.5}$$

Differentiating this with respect to $q_1(\theta)$ yields the following first order condition:

$$\begin{aligned}\beta\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} S'(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon-\theta+(1-\beta)\int_{\underline{\varepsilon}}^{\varepsilon(\theta)^*} S'(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon \\ +(1-\beta)\int_{\varepsilon(\theta)^*}^{\bar{\varepsilon}} \theta f(\varepsilon)d\varepsilon \\ -(1-\beta)\frac{\partial\varepsilon(\theta)^*}{\partial q_1}\Big(S(\phi(\theta))-S\big(q_1(\theta)-\varepsilon(\theta)^*\big)+\theta\big(\phi(\theta)-q_1(\theta)+\varepsilon(\theta)^*\big)f(\varepsilon^*)\Big)=0\end{aligned} \tag{A1.6}$$

By definition of $\varepsilon(\theta)^*$ the last term of the left-hand side is zero and the other "non-integral" terms cancel and I am left with

$$\begin{aligned}\beta\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} S'(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon-\theta+(1-\beta)\int_{\underline{\varepsilon}}^{\varepsilon(\theta)^*} S'(q_1(\theta)-\varepsilon)f(\varepsilon)d\varepsilon \\ +(1-\beta)\int_{\varepsilon(\theta)^*}^{\bar{\varepsilon}} \theta f(\varepsilon)d\varepsilon=0\end{aligned} \tag{A1.7}$$

Rearranging this yields

$$E\big[S'(q_1(\theta)-\varepsilon)\big]-\theta=(1-\beta)\int_{\varepsilon(\theta)^*}^{\bar{\varepsilon}}\big[S'(q_1(\theta)-\varepsilon)-\theta\big]f(\varepsilon)d\varepsilon \tag{A1.8}$$

Q.E.D.

Proof of Proposition 2: Before proving Proposition 2, I will state and prove two lemmas that will simplify the maximization problem. Under adverse selection and full commitment, the contract offered by the principal must satisfy the following participation and incentive constraints:

$$E\big[\underline{t}(\varepsilon)-\underline{\theta}\big(\underline{q}_1+(1-\beta)\underline{q}_2(\varepsilon)\big)\big]\geq 0 \tag{A1.9}$$

$$E\big[\bar{t}(\varepsilon)-\bar{\theta}\big(\bar{q}_1+(1-\beta)\bar{q}_2(\varepsilon)\big)\big]\geq 0 \tag{A1.10}$$

$$E\big[\underline{t}(\varepsilon)-\underline{\theta}\big(\underline{q}_1+(1-\beta)\underline{q}_2(\varepsilon)\big)\big]\geq E\big[\bar{t}(\varepsilon)-\underline{\theta}\big(\bar{q}_1+(1-\beta)\bar{q}_2(\varepsilon)\big)\big] \tag{A1.11}$$

$$E\big[\bar{t}(\varepsilon)-\bar{\theta}\big(\bar{q}_1+(1-\beta)\bar{q}_2(\varepsilon)\big)\big]\geq E\big[\underline{t}(\varepsilon)-\bar{\theta}\big(\underline{q}_1+(1-\beta)\underline{q}_2(\varepsilon)\big)\big] \tag{A1.12}$$

Lemma A1: The efficient type's participation constraint, (A1.10), can be ignored.

Proof: The incentive constraint of the efficient type gives me $E\big[\underline{t}(\varepsilon)-\underline{\theta}\big(\underline{q}_1+(1-\beta)\underline{q}_2(\varepsilon)\big)\big]\geq E\big[\bar{t}(\varepsilon)-\underline{\theta}\big(\bar{q}_1+(1-\beta)\bar{q}_2(\varepsilon)\big)\big]$. Since $\bar{\theta}\geq\underline{\theta}$, this implies $E\big[\underline{t}(\varepsilon)-\underline{\theta}\big(\underline{q}_1+(1-\beta)\underline{q}_2(\varepsilon)\big)\big]\geq E\big[\bar{t}(\varepsilon)-\bar{\theta}\big(\bar{q}_1+(1-\beta)\bar{q}_2(\varepsilon)\big)\big]$. From the participation contraint of the inefficient type, I obtain thus the participation constraint of the efficient type satisfied with strict inequality. Q.E.D.

Lemma A2: The incentive constraint of the inefficient type, (A1.11), can be ignored as long as $\underline{q}_1+(1-\beta)E\big[\underline{q}_2(\varepsilon)\big]\geq\bar{q}_1+(1-\beta)E\big[\bar{q}_2(\varepsilon)\big]$ is satisfied.

Proof: Adding the two incentive constraints, (A1.11) and (A1.12), together with the fact that $\bar{\theta} \geq \underline{\theta}$ yields the result. Q.E.D.

Proof of Proposition 2: The constraints affect the objective function negatively. It is thus optimal to satisfy the two remaining constraints with equality. Substituting t in the maximization problem with the transfer obtained from the binding constraints, the problem simplifies to:

$$\begin{aligned}\max_{\underline{q}_1,\bar{q}_1} \nu \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big[& \beta S(\underline{q}_1 - \varepsilon) + (1-\beta) S\big(\underline{q}_1 + \underline{q}_2(\varepsilon) - \varepsilon\big) \\ & - \underline{\theta}\big(\underline{q}_1 + (1-\beta)\underline{q}_2(\varepsilon)\big) - \Delta\theta\big(\bar{q}_1 + (1-\beta)\bar{q}_2(\varepsilon)\big)\Big] f(\varepsilon) d\varepsilon \\ & + (1-\nu) \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Big[\beta S(\bar{q}_1 - \varepsilon) + (1-\beta) S\big(\bar{q}_1 + \bar{q}_2(\varepsilon) - \varepsilon\big) \\ & - \bar{\theta}\big[\bar{q}_1 + (1-\beta)\bar{q}_2(\varepsilon)\big]\Big] f(\varepsilon) d\varepsilon \end{aligned} \tag{A1.13}$$

Proceeding by backward induction, I first considering what happens in the second period. Under full commitment and adverse selection, q_2 will depend on q_1 and the realization of ε since it is implemented when both these variables are realized and observed. Therefore optimizing (A1.13) with respect to q_2 yields

$\underline{q}_2 = \max\{\phi(\underline{\theta}) - \underline{q}_1 + \varepsilon, 0\}$

$\bar{q}_2 = \max\{\phi(\bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta) - \bar{q}_1 + \varepsilon, 0\}$

Taking into account what will happen in the second period when choosing the period-one q, the principal's first-period problem can be written:

$$\begin{aligned} \max_{\underline{q}_1,\bar{q}_1} \nu \Big\{ & \beta \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} S(\underline{q}_1 - \varepsilon) f(\varepsilon) d\varepsilon - \underline{\theta}\underline{q}_1 + (1-\beta) \int_{\underline{\varepsilon}}^{\varepsilon^*} S(\underline{q}_1 - \varepsilon) f(\varepsilon) d\varepsilon \\ & + (1-\beta) \int_{\varepsilon^*}^{\bar{\varepsilon}} \{S\big(\underline{q}_1 + \underline{q}_2(\varepsilon) - \varepsilon\big) - \underline{\theta}\underline{q}_2(\varepsilon)\} f(\varepsilon) d\varepsilon - \Delta\theta \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \big(\bar{q}_1 + (1-\beta)\bar{q}_2(\varepsilon)\big) f(\varepsilon) d\varepsilon \Big\} \\ & + (1-\nu) \Big\{ \beta \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} S(\bar{q}_1 - \varepsilon) f(\varepsilon) d\varepsilon - \bar{\theta}\bar{q}_1 + (1-\beta) \int_{\underline{\varepsilon}}^{\varepsilon^*} S(\bar{q}_1 - \varepsilon) f(\varepsilon) d\varepsilon \\ & + (1-\beta) \int_{\varepsilon^*}^{\bar{\varepsilon}} \{S\big(\bar{q}_1 + \bar{q}_2(\varepsilon) - \varepsilon\big) - \bar{\theta}\bar{q}_2(\varepsilon)\} f(\varepsilon) d\varepsilon \Big\} \end{aligned}$$

The first order condition with respect to $\underline{q}_1$ writes[22]

$$\begin{aligned} \beta \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} S'(\underline{q}_1 - \varepsilon) f(\varepsilon) d\varepsilon - \underline{\theta} + (1-\beta) \int_{\underline{\varepsilon}}^{\underline{q}_1 - \phi(\underline{\theta})} S'(\underline{q}_1 - \varepsilon) f(\varepsilon) d\varepsilon & \\ + (1-\beta) \int_{\underline{q}_1 - \phi(\underline{\theta})}^{\bar{\varepsilon}} \underline{\theta} f(\varepsilon) d\varepsilon = 0 & \end{aligned} \tag{A1.14}$$

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[22] When neglecting the terms that disappear or are equal to zero.

Simplifying this expression yields

$$E\big[S'(\underline{q}_1-\varepsilon)\big]-\underline{\theta}=(1-\beta)\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\big[S'(\underline{q}_1-\varepsilon)-\underline{\theta}\big]f(\varepsilon)d\varepsilon \tag{A1.15}$$

The output for type $\underline{\theta}$ remains at the first-best level.

Turning now to $\bar{\theta}$, I obtain the following first order condition[23]

$$\beta\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}S'(\bar{q}_1-\varepsilon)f(\varepsilon)d\varepsilon-\bar{\theta}+(1-\beta)\int_{\underline{\varepsilon}}^{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}S'(\bar{q}_1-\varepsilon)f(\varepsilon)d\varepsilon$$

$$+(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}^{\bar{\varepsilon}}\bar{\theta}f(\varepsilon)d\varepsilon-\frac{\nu}{1-\nu}\Delta\theta(1-(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}^{\bar{\varepsilon}}f(\varepsilon)d\varepsilon)=0 \tag{A1.16}$$

$$\Leftrightarrow E\big[S'(\bar{q}_1-\varepsilon)\big]-\bar{\theta}-\frac{\nu}{1-\nu}\Delta\theta=(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}^{\bar{\varepsilon}}\big[S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{\nu}{1-\nu}\Delta\theta\big]f(\varepsilon)d\varepsilon \tag{A1.17}$$

The output of $\bar{\theta}$ is distorted downward to prevent the efficient type from mimicking the inefficient type.

From the binding constraints I obtain the following transfers:

$$\underline{t}=\underline{q}_1+(1-\beta)E\big[\underline{q}_2(\varepsilon)\big]+\Delta\theta\big(\bar{q}_1+(1-\beta)E\big[\bar{q}_2(\varepsilon)\big]\big) \tag{A1.18}$$

$$\bar{t}=\bar{\theta}\big(\bar{q}_1+(1-\beta)E\big[\bar{q}_2(\varepsilon)\big] \tag{A1.19}$$

The efficient type enjoys a strictly positive utility. Q.E.D.

Proof of Proposition 3: In this section the initial contract cannot be contingent on ε. Replacing the value of the transfers in the objective functions yields the following unconstrained maxi-

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[23] Still ignoring the terms that will disappear.

mization problem

$$\begin{aligned}
&\max_{\left\{x,\underline{q}_1,\bar{q}_1\right\}} x\nu\Bigg[\int_{\underline{\varepsilon}}^{\underline{q}_1-\phi(\underline{\theta})}\left\{S(\underline{q}_1-\varepsilon)-\underline{\theta}\underline{q}_1\right\}f(\varepsilon)d\varepsilon \\
&+\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\left\{\beta S(\underline{q}_1-\varepsilon)+(1-\beta)S(\phi(\underline{\theta}))-\underline{\theta}\underline{q}_1-(1-\beta)\underline{\theta}(\phi(\underline{\theta})+\varepsilon-\underline{q}_1)\right\}f(\varepsilon)d\varepsilon \\
&-\Delta\theta\bar{q}_1-(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Delta\theta(\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)+\varepsilon-\bar{q}_1)f(\varepsilon)d\varepsilon\Bigg] \\
&(1-x)\nu\Bigg[\int_{\underline{\varepsilon}}^{\bar{q}_1-\phi(\underline{\theta})}\left\{S(\bar{q}_1-\varepsilon)-\bar{\theta}\bar{q}_1\right\}f(\varepsilon)d\varepsilon \\
&+\int_{\bar{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\left\{\beta S(\bar{q}_1-\varepsilon)+(1-\beta)S(\phi(\underline{\theta}))-\bar{\theta}\bar{q}_1-(1-\beta)\underline{\theta}(\phi(\underline{\theta})+\varepsilon-\bar{q}_1)\right\}f(\varepsilon)d\varepsilon \\
&-(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Delta\theta(\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)+\varepsilon-\bar{q}_1)f(\varepsilon)d\varepsilon\Bigg] \\
&(1-\nu)\Bigg[\int_{\underline{\varepsilon}}^{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}\left\{S(\bar{q}_1-\varepsilon)-\bar{\theta}\bar{q}_1\right\}f(\varepsilon)d\varepsilon \\
&+\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Big\{\beta S(\bar{q}_1-\varepsilon)+(1-\beta)S(\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)) \\
&-\bar{\theta}\bar{q}_1-(1-\beta)\bar{\theta}(\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)+\varepsilon-\bar{q}_1)\Big\}f(\varepsilon)d\varepsilon\Bigg]
\end{aligned}\tag{A1.20}$$

The first-order condition of this with respect to $\underline{q}_1$ is[24]

$$\begin{aligned}
&x\nu\Bigg[\int_{\underline{\varepsilon}}^{\underline{q}_1-\phi(\underline{\theta})}\left\{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon \\
&+\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\left\{\beta S'(\underline{q}_1-\varepsilon)-\underline{\theta}+(1-\beta)\underline{\theta}\right\}d\varepsilon\Bigg]=0
\end{aligned}\tag{A1.21}$$

which simplifies to

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\left\{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon=(1-\beta)\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\left\{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon \tag{A1.22}$$

Or equivalently,

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\left\{S'(\underline{q}_1-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon=(1-\beta)\int_{\underline{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\int_{\phi(\underline{\theta})}^{\underline{q}_1-\varepsilon}S''(x)dxf(\varepsilon)d\varepsilon \tag{A1.23}$$

The efficient type's output is not distorted away from its first-best level.

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[24] Terms that disappear are ignored.

The first-order condition with respect to $\bar{q}_1$ is[25]

$$\begin{aligned} x\nu\Big[-\Delta\theta+(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Delta\theta f(\varepsilon)d\varepsilon\Big]+(1-x)\nu\Big[\int_{\underline{\varepsilon}}^{\bar{q}_1-\phi(\underline{\theta})}\big\{S'(\bar{q}_1-\varepsilon)-\bar{\theta}\big\}f(\varepsilon)d\varepsilon \\ +\int_{\bar{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\big\{\beta S'(\bar{q}_1-\varepsilon)-\bar{\theta}+(1-\beta)\underline{\theta}\big\}f(\varepsilon)d\varepsilon+(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Delta\theta f(\varepsilon)d\varepsilon\Big] \\ +(1-\nu)\Big[\int_{\underline{\varepsilon}}^{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}\big\{S'(\bar{q}_1-\varepsilon)-\bar{\theta}\big\}f(\varepsilon)d\varepsilon \\ +\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\big\{\beta S'(\bar{q}_1-\varepsilon)-\bar{\theta}+(1-\beta)\bar{\theta}\big\}f(\varepsilon)d\varepsilon=0 \end{aligned} \tag{A1.24}$$

Which is equivalent to

$$\begin{aligned} \big((1-x)\nu+(1-\nu)\big)\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\big\{S'(\bar{q}_1-\varepsilon)-\bar{\theta}\big\}f(\varepsilon)d\varepsilon-x\nu\Delta\theta= \\ (1-\beta)\Big[(1-x)\nu\Big(\int_{\bar{q}_1-\phi(\underline{\theta})}^{\bar{\varepsilon}}\big\{S'(\bar{q}_1-\varepsilon)-\underline{\theta}\big\}f(\varepsilon)d\varepsilon-\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Delta\theta f(\varepsilon)d\varepsilon\Big)+ \\ +(1-\nu)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\big\{S'(\bar{q}_1-\varepsilon)-\bar{\theta}\big\}f(\varepsilon)d\varepsilon-x\nu\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\Delta\theta f(\varepsilon)d\varepsilon\Big] \end{aligned} \tag{A1.25}$$

This again simplifies to

$$\begin{aligned} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\big\{S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{x\nu}{(1-x)\nu+(1-\nu)}\Delta\theta\big\}f(\varepsilon)d\varepsilon= \\ (1-\beta)\Big[\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\big\{S'(\bar{q}_1-\varepsilon)-\bar{\theta}-\frac{x\nu}{(1-x)\nu+(1-\nu)}\Delta\theta\big\}f(\varepsilon)d\varepsilon \\ +\frac{(1-x)\nu}{(1-x)\nu+(1-\nu)}\int_{\bar{q}_1-\phi(\underline{\theta})}^{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}\big\{S'(\bar{q}_1-\varepsilon)-\underline{\theta}\big\}f(\varepsilon)d\varepsilon\Big] \end{aligned} \tag{A1.26}$$

The inefficient type's output is distorted downward.

Denote the principal's utility function for the optimal quantities and transfers W. By replacing the second-period quantities by their expression as a function of $\underline{q}_1$, $\bar{q}_1$ and ε, W only depends on these latter variables and x. I will now try to determine the optimal value of x.

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[25] Terms that disappear are ignored

$\frac{dW}{dx} = \frac{\partial W}{\partial x} + \frac{\partial W}{\partial \bar{q}_1}\frac{\partial \bar{q}_1}{\partial x}$. From the first-order condition for $\bar{q}_1$ I know that $\frac{\partial W}{\partial \bar{q}_1} = 0$. Hence $\frac{dW}{dx} = \frac{\partial W}{\partial x}$.

$$\begin{aligned}
\frac{dW}{dx} = \nu \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} & \{\beta S(\underline{q}_1 - \varepsilon) + (1-\beta) S(\underline{q}_1 + \underline{q}_2(\varepsilon, \underline{\theta}) - \varepsilon) - \underline{\theta}\underline{q}_1 - \Delta\theta \bar{q}_1 \\
& -(1-\beta)\big(\underline{\theta}\underline{q}_2(\varepsilon,\underline{\theta}) + \Delta\theta \bar{q}_2(\varepsilon,\bar{\theta})\big)\} f(\varepsilon) d\varepsilon \\
-\nu \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} & \{\beta S(\bar{q}_1 - \varepsilon) + (1-\beta) S(\bar{q}_1 + \underline{q}_2(\varepsilon, \bar{\theta}) - \varepsilon) - \bar{\theta}\bar{q}_1 \\
& -(1-\beta)\big(\underline{\theta}\underline{q}_2(\varepsilon,\bar{\theta}) + \Delta\theta \bar{q}_2(\varepsilon,\bar{\theta})\big)\} f(\varepsilon) d\varepsilon \\
+(1-\beta)\frac{x\nu^2}{1-\nu}\phi'(\bar{\theta} + \frac{\nu_2}{1-\nu_2}\Delta\theta) & (1 - F(\bar{q}_1 - \phi(\bar{\theta} + \frac{\nu_2}{1-\nu_2}\Delta\theta)(\Delta\theta)^2))
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \frac{dW}{dx} = & \nu \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \{\beta S(\underline{q}_1 - \varepsilon) + (1-\beta) S(\underline{q}_1 + \underline{q}_2(\varepsilon, \underline{\theta}) - \varepsilon) - \underline{\theta}\underline{q}_1 - (1-\beta)\underline{\theta}\underline{q}_2(\varepsilon,\underline{\theta})\} f(\varepsilon) d\varepsilon \\
& -\nu \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \{\beta S(\bar{q}_1 - \varepsilon) + (1-\beta) S(\bar{q}_1 + \underline{q}_2(\varepsilon, \bar{\theta}) - \varepsilon) - \underline{\theta}\bar{q}_1 - (1-\beta)\underline{\theta}\underline{q}_2(\varepsilon,\bar{\theta})\} f(\varepsilon) d\varepsilon \\
& +(1-\beta)\frac{x\nu^2}{1-\nu}\phi'(\bar{\theta} + \frac{\nu_2}{1-\nu_2}\Delta\theta)(1 - F(\bar{q}_1 - \phi(\bar{\theta} + \frac{\nu_2}{1-\nu_2}\Delta\theta)))(\Delta\theta)^2.
\end{aligned}$$

The first integral is the first-best (social) surplus and the second integral is the same surplus for another feasible contract. Hence, for $\beta > 0$, this part of the derivative is always positive.

Furthermore, for $x = 0$, $\frac{dW}{dx}$ is positive. By increasing x, the value of W is increased. Thus $x = 0$ cannot be optimal.

However, since ϕ' is negative, if $\phi'(\bar{\theta})(\Delta\theta)^2$ is too large then $\frac{dW}{dx}(x = 1)$ is negative. Since $\frac{dW}{dx}$ is a continuous function of x, there exists a x_0 such that $\frac{dW}{dx}(x_0) = 0$. In this case some degree of bunching is optimal (i.e. the level $x_0 \in (0,1)$). In the opposite case, when $\phi'(\bar{\theta})(\Delta\theta)^2$ is small enough, full separation is optimal (i.e. $x = 1$) since for all values of x, W is increasing.

When $\beta = 0$, $\frac{\partial W}{\partial x} = 0$, but $\forall \underline{q}_1, \bar{q}_1, W(\underline{q}_1, \bar{q}_1, x) \leq W(0,0,x)$. Thus, in this very particular case, no investment should take place in the first period. This is quite intuitive, since the first period does not count in the objective function. Only the second-period surplus counts and there is no gain from investing earlier (under uncertainty), thus we are again back to a static adverse selection model. Q.E.D.

Proof of Proposition 4: I will show that with the transfers suggested in proposition 4 any Nash equilibrium yields the full-commitment outcome. I first prove the Proposition in the case of Nash-implementation. Then I argue that by making the moves sequential they also imply unique sub-game perfect implementation of the full-commitment outcome.

Consider first the case of the contract designed for $\underline{\theta}$. The only type that might gain when deviating from telling his true type is this type. Assume that he announces his true type (I will later show that by announcing $\bar{\theta}$ he does not improve his situation). In the second period the agents utility, denoted $\underline{U}$ is

$$\underline{U} = S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_a) + \underline{\theta}\underline{q}_2(\varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_p) - (S(\underline{q}_1 + \underline{q}_2(\varepsilon_p) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_p)) \tag{A1.27}$$

Which simplifies to

$$\underline{U} = S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_a) - (S(\underline{q}_1 + \underline{q}_2(\varepsilon_p) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_p)) \tag{A1.28}$$

Note that this expression is maximized (and equal to zero) when $\varepsilon_a = \varepsilon_p$.

The corresponding gain for the principal, denoted W, is

$$W = S(\underline{q}_1 + \underline{q}_2(\varepsilon_p) - \varepsilon) - \big(S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_a) + \underline{\theta}\underline{q}_2(\varepsilon_p)\big) + S(\underline{q}_1 + \underline{q}_2(\varepsilon_p) - \varepsilon_p) - \underline{\theta}\underline{q}_2(\varepsilon_p) \tag{A1.29}$$

If $\varepsilon_a \neq \varepsilon_p$, the agent will have an incentive to deviate to ε_p to obtain $\underline{U} = 0$[26].

If $\varepsilon_a = \varepsilon_p \neq \varepsilon$, the principal prefers deviating to $\varepsilon_p^{'} = \varepsilon$.
Proof: $W(\varepsilon_p = \varepsilon, \varepsilon_a) > W(\varepsilon_p = \varepsilon_a, \varepsilon_a)$

$$\begin{aligned}
&\Leftrightarrow S(\underline{q}_1 + \underline{q}_2(\varepsilon) - \varepsilon) - \big(S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon) - \underline{\theta}\underline{q}_2(\varepsilon_a) + \underline{\theta}\underline{q}_2(\varepsilon)\big) + S(\underline{q}_1 + \underline{q}_2(\varepsilon) - \varepsilon) - \underline{\theta}\underline{q}_2(\varepsilon) \\
&> S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon) - \big(S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon_a) - \underline{\theta}\underline{q}_2(\varepsilon_a) + \underline{\theta}\underline{q}_2(\varepsilon_a)\big) + S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon_a) - \underline{\theta}\underline{q}_2(\varepsilon_a)
\end{aligned}$$

$$\Leftrightarrow 2\big[S(\underline{q}_1 + \underline{q}_2(\varepsilon) - \varepsilon) - \underline{\theta}\underline{q}_2(\varepsilon)\big] > 2\big[S(\underline{q}_1 + \underline{q}_2(\varepsilon_a) - \varepsilon) - \underline{\theta}\underline{q}_2(\varepsilon_a)\big]$$

Which is true by construction[27].

Finally, it is easy to see that $\varepsilon_a = \varepsilon_p = \varepsilon$ is a Nash equilibrium (no profitable unilateral deviations possible).

Consider now the case of type $\bar{\theta}$

$$\begin{aligned}
\bar{U} = {} & S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a) + \bar{\theta}\bar{q}_2(\varepsilon_p) \\
& - \bar{\theta}\bar{q}_2(\varepsilon_p) - \big(S(\bar{q}_1 + \bar{q}_2(\varepsilon_p) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_p) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_p)\big)
\end{aligned} \tag{A1.30}$$

Which simplifies to

$$\begin{aligned}
\bar{U} = {} & S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a) \\
& - \big(S(\bar{q}_1 + \bar{q}_2(\varepsilon_p) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_p) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_p)\big)
\end{aligned} \tag{A1.31}$$

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[26] Note that if ε_p is such that the optimal q_2 is equal to zero, then the agent can announce any ε_a such that $q_2(\varepsilon_a) = 0$ at the Nash equlibrium, but the resulting outcome (quantity and transfer) will be the same (equal to zero).

[27] Unless ε_a and ε are different but such that they yield a quantity equal to zero. But again, this is the optimal outcome. From now, I will ignore these possibilities since they also implement the full commitment outcome.

Again note that this expression is maximized (and equal to zero) when $\varepsilon_a = \varepsilon_p$.

The corresponding gain for the principal, denoted W, is

$$W = S(\bar{q}_1 + \bar{q}_2(\varepsilon_p) - \varepsilon) - \big(S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a)$$
$$+\bar{\theta}\bar{q}_2(\varepsilon_p)\big) + S(\bar{q}_1 + \bar{q}_2(\varepsilon_p) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_p) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_p) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_p) \quad \text{(A1.32)}$$

where the last term comes from the efficient type's rent.

If $\varepsilon_a \neq \varepsilon_p$, the agent will have an incentive to deviate to ε_p to obtain $\bar{U} = 0$.

If $\varepsilon_a = \varepsilon_p \neq \varepsilon$, the principal prefers deviating to $\varepsilon_p^{'} = \varepsilon$.
Proof: $W(\varepsilon_p = \varepsilon, \varepsilon_a) > W(\varepsilon_p = \varepsilon_a, \varepsilon_a)$

$$\Leftrightarrow S(\bar{q}_1 + \bar{q}_2(\varepsilon) - \varepsilon) - \big(S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a)$$
$$+\bar{\theta}\bar{q}_2(\varepsilon)\big) + S(\bar{q}_1 + \bar{q}_2(\varepsilon) - \varepsilon) - \bar{\theta}\bar{q}_2(\varepsilon) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon)$$
$$> S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon) - \big(S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_a) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a)$$
$$+\bar{\theta}\bar{q}_2(\varepsilon_a)\big) + S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_a) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a)$$

$$\Leftrightarrow 2\big[S(\bar{q}_1 + \bar{q}_2(\varepsilon) - \varepsilon) - \bar{\theta}\bar{q}_2(\varepsilon) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon)\big] > 2\big[S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a)\big]$$

Again this is true by construction.

Finally, it is easy to see that $\varepsilon_a = \varepsilon_p = \varepsilon$ is a Nash equilibrium (no profitable unilateral deviations possible).

To complete the proof, consider what happens if $\underline{\theta}$ claims to be of type $\bar{\theta}$. In this case he will obtain the following utility.

$$\bar{U}(\underline{\theta}) = S(\bar{q}_1 + \bar{q}_2(\varepsilon_a) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_a) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_a) + (\bar{\theta} - \underline{\theta})\bar{q}_2(\varepsilon_p)$$
$$-\big(S(\bar{q}_1 + \bar{q}_2(\varepsilon_p) - \varepsilon_p) - \bar{\theta}\bar{q}_2(\varepsilon_p) - \frac{\nu}{1-\nu}\Delta\theta\bar{q}_2(\varepsilon_p)\big) \quad \text{(A1.33)}$$

The term $(\bar{\theta} - \underline{\theta})\bar{q}_2(\varepsilon_p)$ does not depend on the agent's action. So for the same reason as in previous discussions, the best the agent can do is to announce $\varepsilon_a = \varepsilon_p$.

Note that the scheme described in this section only leads to Nash implementation of the full-commitment outcome. Sub-game perfect Nash implementation would be a further refinement of this mechanism. A way of obtaining such a result in this model is to consider sequential announcements of ε instead of simultaneous announcements as considered previously. The previous mechanism and transfers could be modified to implement the full-commitment outcome as a sub-game perfect Nash equilibrium in a sequential two-step game. In a first step the agent announces the value of ε and the principal agrees or challenges this announcement.

If the principal agrees, the full-commitment quantity, $q_2(\varepsilon_a)$ is implemented and the transfers described in Proposition 4 (with $\varepsilon_a = \varepsilon_p$) are paid. If the principal challenges the agent's announcement he announces another value of ε which the agent then confirms or challenges. In both cases the quantity and transfers are those described in the simultaneous game and the true ε will be announced at equilibrium. The proof of Proposition 4 is easily adjusted to account for this sequential variant of the game and the same outcome as under full commitment and verifiability of ε is a (unique) sub-game perfect Nash equilibrium. Q.E.D.

Proof of Proposition 5: Recall that under complete contracting these investments are given by

- $\bar{q}_1^c$ is such that

$$E\left[S'(\bar{q}_1^c-\varepsilon)\right]-\bar{\theta}-\frac{\nu}{1-\nu}\Delta\theta-(1-\beta)\int_{\bar{q}_1^c-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}^{\bar{\varepsilon}}\left[S'(\bar{q}_1^c-\varepsilon)-\bar{\theta}-\frac{\nu}{1-\nu}\Delta\theta\right]f(\varepsilon)d\varepsilon=0. \tag{A1.34}$$

- $\bar{q}_2^c(\varepsilon)=\max\{\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)-\bar{q}_1^c+\varepsilon;0\}$.

and under incomplete contracting by

- $\bar{q}_1^i$ is such that

$$\begin{aligned}
&\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\left\{S'(\bar{q}_1^i-\varepsilon)-\bar{\theta}-\frac{x\nu}{(1-x)\nu+(1-\nu)}\Delta\theta\right\}f(\varepsilon)d\varepsilon\\
-(1-\beta)\Bigg[&\int_{\bar{q}_1^i-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}^{\bar{\varepsilon}}\left\{S'(\bar{q}_1^i-\varepsilon)-\bar{\theta}-\frac{x\nu}{(1-x)\nu+(1-\nu)}\Delta\theta\right\}f(\varepsilon)d\varepsilon\\
&+\frac{(1-x)\nu}{(1-x)\nu+(1-\nu)}\int_{\bar{q}_1^i-\phi(\underline{\theta})}^{\bar{q}_1^i-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)}\left\{S'(\bar{q}_1^i-\varepsilon)-\underline{\theta}\right\}f(\varepsilon)d\varepsilon\Bigg]=0
\end{aligned} \tag{A1.35}$$

  where $x\in(0,1]$.

- $\bar{q}_2^i(\varepsilon)=\max\{\phi(\bar{\theta})-\bar{q}_1^i+\varepsilon;0\}$.

First I will prove that $\bar{q}_1^i>\bar{q}_1^c$. Then I will use this to show that $\bar{q}^i=\bar{q}_1^i+\bar{q}_2^i(\varepsilon)>\bar{q}^c=\bar{q}_1^c+\bar{q}_2^c(\varepsilon)$.

I cannot directly compare the value of $\bar{q}_1^i$ and $\bar{q}_1^c$ since they are given by fairly complex implicit expressions. I therefore proceed in two steps. First I show that for $\bar{q}_1^i$, the left-hand side of equation A1.34 is negative. The second step consists of showing that the left-hand side of equation A1.34 is a decreasing function of $\bar{q}_1$. These two facts allow me to conclude that $\bar{q}_1^i>\bar{q}_1^c$.

Notice that since $S''<0$, $\phi=(S')^{-1}$ and $\nu\in(0,1)$, it is easy to show that

$$\phi(\bar{\theta})\geq\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2}\Delta\theta)>\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta). \tag{A1.36}$$

The left-hand side of the expression of equation A1.34 (hereafter denoted $LHS^c(\bar{q}_1)$) evaluated at $\bar{q}_1^c$ takes the following value

$$\begin{aligned}
LHS^c(\bar{q}_1^c) = \quad & \Delta\theta\left[1-(1-\beta)(1-F(\bar{q}_1^c-\phi(\bar{\theta}+\tfrac{\nu}{1-\nu}\Delta\theta))\right]\left[\tfrac{x\nu}{(1-x)\nu+1-\nu}-\tfrac{\nu}{1-\nu}\right] \\
& -(1-\beta)\tfrac{x\nu}{(1-x)\nu+1-\nu}\Delta\theta\left[F(\bar{q}_1^c-\phi(\bar{\theta}+\tfrac{\nu}{1-\nu}\Delta\theta))-F(\bar{q}_1^c-\phi(\bar{\theta}+\tfrac{\nu_2}{1-\nu_2}\Delta\theta))\right] \\
& -(1-\beta)\int_{\bar{q}_1^c-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2})}^{\bar{q}_1^c-\phi(\bar{\theta}+\frac{\nu}{1-\nu})}\left[S'(\bar{q}_1^c-\varepsilon)-\bar{\theta}\right]f(\varepsilon)d\varepsilon \\
& -(1-\beta)\tfrac{(1-x)\nu}{(1-x)\nu+1-\nu}\int_{\bar{q}_1^c-\phi(\underline{\theta})}^{\bar{q}_1^c-\phi(\bar{\theta}+\frac{\nu_2}{1-\nu_2})}\left[S'(\bar{q}_1^c-\varepsilon)-\underline{\theta}\right]f(\varepsilon)d\varepsilon
\end{aligned}$$

It can easily be verified that all these terms are less than zero.

Furthermore the derivative of $LHS^c(\bar{q}_1)$ is given by the following expression

$$\begin{aligned}
\frac{dLHS^c(\bar{q}_1)}{d\bar{q}_1} &= E\left[S''(\bar{q}_1-\varepsilon)\right]-(1-\beta)\int_{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)}^{\bar{\varepsilon}} S''(\bar{q}_1-\varepsilon)f(\varepsilon)d\varepsilon \\
&= \beta E\left[S''(\bar{q}_1-\varepsilon)\right]+(1-\beta)\int_{\underline{\varepsilon}}^{\bar{q}_1-\phi(\bar{\theta}+\frac{\nu}{1-\nu}\Delta\theta)} S''(\bar{q}_1-\varepsilon)f(\varepsilon)d\varepsilon
\end{aligned}$$

All the terms after the last equality sign are negative and therefore I can conclude that $\bar{q}_1^i > \bar{q}_1^c$.

To prove that $\bar{q}^i = \bar{q}_1^i + \bar{q}_2^i(\varepsilon) > \bar{q}^c = \bar{q}_1^c + \bar{q}_2^c(\varepsilon)$ I will consider the four possible scenarios according to the value of ε and the corresponding second-period investments[28].

1. $\bar{q}_2^c(\varepsilon) = 0$ and $\bar{q}_2(\varepsilon)^i = 0$: In this case it is obvious that $\bar{q}^c = \bar{q}_1^c < \bar{q}^i = \bar{q}_1^i$.
2. $\bar{q}_2^c(\varepsilon) = 0$ and $\bar{q}_2^i(\varepsilon) > 0$: Similarly, it is straightforward that $\bar{q}^c = \bar{q}_1^c < \bar{q}_1^i < \bar{q}_1^i + \bar{q}_2^i(\varepsilon) = \bar{q}^i$.
3. $\bar{q}_2^c(\varepsilon) > 0$ and $\bar{q}_2^i(\varepsilon) = 0$: From the FOCs of the second-period investment I know that

   $$S'(\bar{q}_1^c + \bar{q}_2^c(\varepsilon) - \varepsilon) = \bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta$$

   and

   $$S'(\bar{q}_1^i - \varepsilon) < \bar{\theta}.$$

   Since $\bar{\theta} < \bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta$ and $S'' < 0$, I obtain $\bar{q}^i = \bar{q}_1^i > \bar{q}^c = \bar{q}_1^c + \bar{q}_2^c(\varepsilon)$.
4. $\bar{q}_2^c(\varepsilon) > 0$ and $\bar{q}_2^i(\varepsilon) > 0$: From the FOCs of the second-period investment I know that

   $$S'(\bar{q}_1^c + \bar{q}_2^c(\varepsilon) - \varepsilon) = \bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta$$

   and

   $$S'(\bar{q}_1^i + \bar{q}_2^i(\varepsilon) - \varepsilon) = \bar{\theta}.$$

   Using the same techniques as above I obtain $\bar{q}^i = \bar{q}_1^i + \bar{q}_2^i(\varepsilon) > \bar{q}^c = \bar{q}_1^c + \bar{q}_2^c(\varepsilon)$.

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[28] If I knew that for all ε $\bar{q}_2^i(\varepsilon) \geq \bar{q}_2^c(\varepsilon)$ the result would be immediate. However, so far I have been unable to prove this.

I can therefore conclude that for all possible values of ε, $\bar{q}^c < \bar{q}^i$.

I also need to show that $\bar{q}^{FB}$ is always greater (or equal) than $\bar{q}^i$ (and therefore also greater than $\bar{q}^c$). The technique used above can be applied to show that $\bar{q}_1^{FB} \geq \bar{q}_1^i$ and then that $\bar{q}^{FB} = \bar{q}_1^{FB} + \bar{q}_2^{FB}(\varepsilon) \geq \bar{q}^i = \bar{q}_1^i + \bar{q}_2^i(\varepsilon)$.

Q.E.D.

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